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F. PUNGA

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BY

FRANKLIN PUNGA

TRANSLATED FROM THE GERMAN BY

R. F. LOOSER, A.M.I.E.E.

WITH EIGHTY-ONE ILLUSTRATIONS

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PREFACE

A TREATISE on single-phase motors is largely a treatise on the sparking problem, for a thorough knowledge of the phenomenon of commutation is of even more importance in connection with single-phase than in connection with continuous-current motors.

Electrical science is particularly indebted to Messrs Parshall and Hobart, the authors of *Electric Generators*, and to Prof. E. Arnold, the author of *Die Gleichstrommaschine*, for investigating and elucidating the problem of sparking. Whilst Messrs Parshall and Hobart have evolved a simple and eminently practical criterion for predetermining sparking, Prof. Arnold has overcome the difficulties connected with a close mathematical treatment on this difficult subject.

Amongst the other investigators who have contributed to our present knowledge, the names of Thornburne Reid, Pichelmayer, Rothert, Fischer-Hinnen, Prof. G. Kapp, and Niethammer may be mentioned.

The method of treating this subject, set forth in the pages of this book, was first outlined by the author in the *Zeitschrift für Elektrotechnik*, Vienna, 1902, Nos. 30, 31, and 32, and, although his con-

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tribution can only be small compared with that of the other investigators referred to, yet he ventures to hope that some of the results obtained, which appear new, may interest and benefit the profession at large.

The present book is practically a translation of the German original, with the addition of a second appendix containing a discussion on some interesting oscillograph tests relating to the commutation of single-phase motors. The oscillograms in question, hitherto unpublished, were kindly placed at my disposal by Mr F. Creedy, to whom I also wish to express my thanks for valuable suggestions and for assistance in correcting the proofs.

In the opening chapters, the origin and nature of sparking are gone into in detail; the manner of calculating the reactance voltage is explained, and means of preventing or minimising sparking are discussed at length, particular attention being paid to the method of reducing sparking by inserting resistances in the commutator leads. A convenient method of computing the reaction of the commutating coils upon the field flux is given in Chapter VI. This reaction is by no means always insignificant—as is shown in an example—and has hitherto hardly received the attention it deserves.

It is not claimed that Chapters VIII., IX., and X., which deal with the general theory of single-phase commutator motors, are perfect theoretically; indeed, these chapters are merely intended to lead up to Chapters XII., XIII., and XIV., containing the final conclusions. In Chapter XV., a series motor, repulsion motor, and compensated repulsion motor are

calculated through, the calculations being based on equal manufacturing costs in each case.

The detailed theoretical investigation of the repulsion motor has been reserved for the appendix so as not to discourage those readers who are not mathematically inclined.

The design of the single-phase series motor has been more specifically dealt with by Messrs Lamme and Finsi, while Heubach was one of the foremost to expound its theory; Eichberg, Winter, Latour, Osnos, Danielson, Pichelmayer and Niethammer have been prominent in connection with the compensated repulsion motor, while C. P. Steinmetz has largely contributed to our knowledge of the repulsion motor. One of the most illuminating treatises on single-phase motors generally is that contributed by Prof. Görges to the *Elektrotechnische Zeitschrift*, 1903, No. 15.

The single-phase motor invented by Eichberg, Winter and Latour is often referred to as the compensated series motor, and thereby an impression is created that this motor is a series motor provided with a coil for compensating the self-induction. Danielson was, perhaps, the first to call it a compensated repulsion motor, and in this he was undoubtedly right. The reason for designating this motor a series motor with a compensating winding must be ascribed to an imperfect understanding of its nature.

In a future edition of this book, it is intended also to include those single-phase motors which do not possess a 'series' characteristic. Though of scarcely any importance in connection with traction

work, the 'constant-speed' type of single-phase motor is nevertheless destined to play a prominent part in the future, notably in England, where single-phase systems are still plentiful.

To my friend, Mr R. F. Looser, I am greatly indebted for the trouble he has taken in translating this book from the German, and for the expert and clear manner in which he has carried the task through.

F. PUNGA.

March 1906.

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NOTATIONS

a is the ratio $\frac{\text{armature A.T.}}{\text{field A.T.}}$.

A.T. = ampere turns.

a.c. = alternating current.

$b = \frac{N'}{N}$.

d.c. direct current.

e_1, e_2 denote the voltages between brush and segment.

E.M.F. electromotive force.

E_a is the voltage induced in the commutating coil by the action of the armature flux.

E_f sparking voltage per segment.

E_{fb} sparking voltage per brush.

E_i the voltage induced in the commutating coil.

E_p the voltage induced in the commutating coil by the action of the field flux.

E_r the reactance voltage.

e_u the effective drop of voltage under a brush at imperfect commutation.

e_v the effective drop of voltage under a brush at perfect commutation.

g the effective length of one armature turn.

g' the "free" length of one armature turn.

H_1 the number of stator slots per pole.

H_2 the number of rotor slots per pole.

$H = \frac{H_1 + H_2}{2}$.

l the effective length of the armature.

M the flux per pole.

M.M.F. magnetomotive force.

n the number of segments covered by one brush.

N the frequency of the alternating supply.

$N' = \frac{pU}{60}$ the frequency of rotation.

p the number of pole-pairs.

P.D. potential difference.

$q = v \times x$.

s the number of turns in series between the brushes.

T the duration (in seconds) of the short-circuit.

U revolutions per minute.

W the contact resistance between all the brushes of one set and the commutator.

z the number of turns per segment.

α the displacement (in degrees) of the brushes from the stator-axis referred to two poles.

$\beta = 0{\cdot}955 \frac{N'}{N} (2\lambda - \lambda^2)$.

δ the length of air-gap.

λ the displacement of the brushes from the stator-axis ; the maximum displacement occurs when $\lambda = 1$; $\alpha = \lambda \times 90$ degrees.

$x = 1 - 1{\cdot}5\lambda^2 + 0{\cdot}5\lambda^3$.

$\pi = 3{\cdot}1415926$.

1 inch $= 2{\cdot}54$ cm.

1 lb. $= 0{\cdot}45$ kg.

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INTRODUCTION

If the current supplied to the terminals of any ordinary continuous-current series motor be reversed in direction, the motor will still continue to rotate in the same sense, because the armature current and the field current have changed their directions simultaneously. It follows that any continuous-current series motor of usual design will run with alternating current, and, if the frequency of the alternating supply be sufficiently low, the motor will operate almost exactly like its continuous-current prototype. But as the supply frequency increases, the effects peculiar to alternating currents—viz. capacity and induction effects—manifest themselves more and more. The effects due to the capacity of the armature and field circuits, even at the highest frequencies employed in practice, are never serious, and they will therefore not be further considered. The effects of self-induction, on the other hand, are of great importance, since they lead to a phase-displacement between current and voltage, thus increasing the apparent watts' input and the current input. Another very important question in connection with single-phase commutator motors relates to the sparking at the commutator.

In the following chapters these two cardinal problems will be gone into in detail; but it is not intended to enlarge the book unnecessarily by including an elementary theory of electro-magnetism or the theory of armature windings and so forth, as these subjects have been dealt with in many excellent books on continuous-current dynamos. A list of such books is appended.

The clearest and quickest way of solving the sparking problem of single-phase commutator motors is to start with the theory—now well understood—of the commutation of d.c. machines and to extend and modify this theory to suit the special conditions of a.c. working.

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PART I.—THEORY.

CHAPTER I.

THE ORIGIN OF SPARKING.

DURING the greater part of a revolution the only current flowing in any one armature coil is the armature current J, but when the coil becomes short-circuited by the brushes a short-circuit current, C, also tends to flow in the coil in question. We may imagine that these two currents are superposed. It will be clear from fig. 1—in which the dotted line a, b, c, d, e_2 and e_1 indicates the short-circuit—that their directions coincide in one segment but are opposite in the other. The consequence is an unequal distribution of current under the brushes from segment to segment, *this unequal distribution of current being the direct cause of sparking.*

According to Ohm's law, the drop of pressure at any point of the contact area between brush and commutator is equal to the current at that point

multiplied by the contact resistance. If, for a given quality of brush, the specific contact resistance be assumed to be a constant quantity, then the voltage drop at any point of the contact area will be proportional to the current density at that point. In order to arrive at the distribution of current under the brushes, all that is necessary, therefore, is to

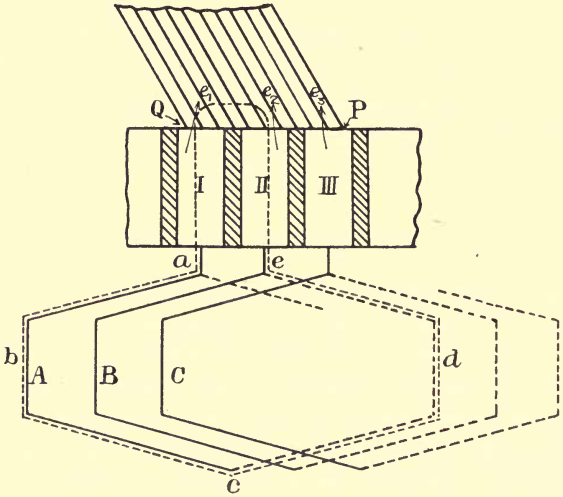

FIG. 1.

determine the distribution of the voltage drop under the brushes. This we will do by considering seriatim the voltages induced in the short-circuited coil.

There is first of all the E.M.F. of induction, or the reactance voltage, as it is usually called in dynamo design. It is caused by the reversal of current which takes place in the coil during the

period of commutation. Before the coil is short-circuited by the brushes it carries a current J. During the period of commutation this current J diminishes, passes through zero value, and grows in the opposite direction until—at the moment the coil is leaving the brushes—it has assumed the value $-J$. Since the current J produces a magnetic flux (M_c) through the coil, it is clear that this flux also reverses during the short period of commutation. The curve representing this change of current or flux with the time is difficult to determine theoretically[1]; but it follows from first principles that, whatever shape this curve may assume, the *mean* reactance voltage is

$$E_r = -\frac{2M_c z}{T} \text{ (in absolute units) or } E_r = -\frac{2M_c z}{T} 10^{-8} \text{ volts,}$$

in which z = number of turns of the coil in question and T = time of short-circuit in seconds. That this equation is correct is at once apparent when it is remembered that the voltage induced in any conductor is equal to the number of lines of force which cut the conductor in unit time. This reactance voltage, as is well known, opposes any *change* of current, and this is the reason why the negative sign is prefixed to the right-hand side of the equation.

Another E.M.F. in the coil is the ohmic drop in pressure. The resistance to be considered in this connection is the sum total of the resistances in the short-circuited coil, in the commutator leads, in the commutator segments, in the brush, and in the brush contact. However, the first four resistances

[1] Such curves have been obtained experimentally with the aid of the oscillograph. See Appendix II.

are usually negligible compared with the last-named resistance; in most cases, therefore, the contact resistance alone need be considered. It was pointed out above that the main current from the brush had to flow through one segment (say, segment I) against the short-circuit current, while the current from the brush to the other segment (II) was augmented by the short-circuit current. Let the resultant currents through segments I and II be denoted by i_1 and i_2 respectively. These two currents cause a drop in voltage across the contact resistance of e_1 volts and e_2 volts. In fig. 1 the directions of these two voltages are indicated by arrows. It is seen that e_1 and e_2 act in opposite directions with regard to the path (dotted) of the short-circuit current—i.e. *only the difference* $e_2 - e_1$ *of these two voltages acts in the short-circuited coil.*

The magnitude of $e_2 - e_1$ varies with the time and is not easy to determine theoretically for every instant; but the *mean* value of $e_2 - e_1$ (called the 'sparking voltage' and denoted by E_f) may be calculated without difficulty.

This 'sparking voltage' is of the utmost importance in dynamo design. If, for instance, $e_2 = e_1$ (*i.e.* $e_2 - e_1 = 0$), then the current through segment II equals that through segment I and the commutation is perfect.

But if e_2 is greater than e_1, then the current in segment II is greater than that in segment I; and *vice versâ.* Sparking will occur if the difference between these two voltages exceeds a certain limit, and it will grow worse with increasing differences.

By way of definition, we will call the sparking

voltage positive when it acts in the same sense in the short-circuited coil as the voltage induced in the coil by the field due to the armature ampere-turns, and negative when it acts in the opposite direction.

There is yet a third voltage in the coil; this we will call 'external' voltage (E_i) since it is produced by external fields, such as the ordinary field flux, the armature field (produced by the armature A.T.) and the fields of auxiliary poles, if any.

It was explained above that the reactance voltage E_r opposes any change in current in the short-circuited coil; but as the reversal of current must perforce take place, it follows that this reactance voltage must be balanced in the coil by a voltage of the same value but of opposite direction. This voltage, therefore, must have the value $-E_r = \frac{2M_c z}{T} 10^{-8}$ volts, and it is obviously the resultant of the two other voltages which appear in the short-circuited coil. Thus we arrive at the following equation, which is the fundamental equation for the commutation of electric machines with commutators:

$$E_i - E_f = -E_r \quad \text{or} \quad E_f = E_r + E_i^{1} \quad . \quad \text{(I.)}$$

Some important conclusions may be drawn from equation I.:

(*a*) $E_i = -E_r$, hence $E_f = 0$;

or, in words, *the current is evenly distributed under the brushes*, i.e. *the commutation is perfect, if the voltage produced in the coil by the external fields is equal in value but opposite in sense to the reactance voltage.*

(*b*) $E_i = 0$, hence $E_f = E_r$;

[1] As a rule, E_i is opposite in sense to E_r.

i.e. *if no E.M.F. is induced in the short-circuited coil by the external fields, then the current distribution under the brushes is such that the mean value of* $e_2 - e_1$ *during the period of commutation is exactly equal to the mean value of the reactance voltage.*

(*c*) If E_i is smaller than $-E_r$

then E_f assumes a negative value, and this means that the current density is greater in segment I than in segment II.

The 'sparking voltage' is a better criterion for the commutation than the reactance voltage, although the latter may be used in most ordinary machines for determining the quality of commutation.

CHAPTER II.

THE CALCULATION OF THE REACTANCE VOLTAGE.

THE reactance voltage has the value $-\frac{2M_c z}{T} 10^{-8}$ volts, and it could be readily calculated if M_c were known. But the value of M_c depends on the shape of the armature teeth, on the number of conductors simultaneously short-circuited, on the number of slots in which these conductors are distributed, on the construction of the field magnets (whether laminated or solid) and on the arrangement and length of the armature end connections. These conditions are difficult to express mathematically, and it has for this reason become customary to calculate the reactance voltage by the approximate method of Messrs

Parshall & Hobart.[1] The approximation lies in the fact that it is assumed that an armature conductor carrying one ampere produces per cm. length a certain number of lines of force. This number varies according to whether the conductor is embedded in the slots or is 'free.' Messrs Parshall and Hobart have found experimentally that, on an average, an embedded conductor produces 4 lines of force per cm. length per ampere, while a 'free' conductor only gives rise to about 0·8 lines of force per cm. length per ampere.

A convenient formula for the reactance voltage, based upon Parshall & Hobart's method and first published by Mavor, is

$$E_r = \frac{4zA(g + 0{\cdot}1g')}{pM} \quad . \quad . \quad . \quad \text{(II.)}.^2$$

in which

z = number of turns per segment;
A = capacity of armature (current × counter-E.M.F.);
g = the effective length of a turn in cm., *i.e.* twice the effective length of the armature core;
g' = free length of a turn in cm. (g' may be taken as three times the pole-pitch);
p = number of pole-pairs and
M = field flux per pole.

The permissible value of E_r depends largely on the

[1] See *Electric Generators*, by Parshall & Hobart.

A more complicated method, which gives somewhat closer results, has been elaborated by Prof. E. Arnold of Carlsruhe.

[2] The proof of this formula will be found in the *Electrical Review* of London for April 22, 1904.

quality of brush used and is determined experimentally for any given set of conditions.

The only quantitative difference between the above formula and that of Hobart is the factor 4. The reason for this is that Hobart assumes the short-circuit current to vary sinusoidally and introduces into his formula the amplitude of the sine wave, whereas our formula relates to the *mean* value of any wave form. Hobart's formula, therefore, gives a value $\frac{\pi}{2}$ times that obtained from equation (II.). When Hobart's formula is employed for determining the relative values of the motor dimensions, etc., to satisfy sparkless commutation, the reactance voltage permissible is $\frac{\pi}{2}$ times that which may be allowed in our case. The final result, of course, is the same whether Hobart's formula be used or the one quoted above.

Formula (II.) is only approximately correct, the reason being that a number of minor effects have not been taken account of. But for one and the same *type* of machine, for which the permissible maximum reactance voltage is known, the formula furnishes a satisfactory and convenient criterion regarding commutation. But for pre-determining the commutation of a different type of machine or for different kinds of brushes, the minor influences referred to have to be taken, at least approximately, into account. These influences may either increase or diminish the reactance voltage. A dynamo, for instance, with very deep and narrow slots will certainly have a higher reactance voltage than another dynamo in which the

slots are wide and shallow. Laminated fields, too, increase the reactance voltage, but there is a lack of data as to the exact amount. Of importance, too, in fixing the proper value of E_r is the distance of the short-circuited coil from the nearest pole-tip. All these minor influences are difficult to investigate mathematically and their approximate evaluation must be left to the designer's judgment.

In cases where the external fields play a prominent part in connection with the short-circuited coil, the reactance voltage forms only a part of the sparking-voltage—see equation (I.)—and the minor influences enumerated above lose still more in importance.

CHAPTER III.

MEANS FOR PREVENTING SPARKING.

If sparking is to be prevented, $E_f\ (=E_r+E_i)$ must be kept below a certain limit; and sparking disappears entirely when $E_f=0$. In other words, the external voltage E_i ought to be kept as nearly as possible equal to $-E_r$ at all loads. $-E_r$ means a voltage equal in magnitude to the reactance voltage but opposite in direction. E_r increases proportionally with the dynamo load and in order, therefore, to fulfil the condition just mentioned, E_i should also increase in the same ratio.

Some years ago this condition was satisfied by shifting the brushes with varying load so as to bring more or less of the field flux into play. The brushes were shifted until the resultant of armature

flux and field flux through the short-circuited coil attained a value and direction such as would prevent sparking. Since the action of the armature flux tends to increase sparking, it follows that the brushes must be moved in such a direction that the field flux thereby brought into play opposes the armature flux. In d.c. generators, therefore, the brushes must be shifted in the direction of rotation, while in motors the brushes must be gradually moved backwards as the load increases. At no-load, the best brush position is the neutral position. This expedient of shifting the brushes is of no avail, however, if the value of the field flux which links with the short-circuited coil when the brushes are under the pole-tips is too small to compensate the reactance voltage sufficiently to prevent sparking.

At the present time, however, d.c. machines are universally required to operate satisfactorily at all loads with the brushes in a fixed (not necessarily the neutral) position. In such cases the only guarantee against sparking is to design the machine in such a way that the reactance voltage—or, better still, the sparking voltage—does not exceed, at the highest load, a certain value. In reversible motors, such as traction motors, the brushes are always in the neutral position.

When the brushes are in a fixed position, there is only one load for which

$$E_i = -E_r \quad \text{or} \quad E_i + E_r = 0.$$

At any other load the value of $E_i + E_r$ differs from zero, being positive for larger loads and negative for smaller loads. Since sparking depends only on the

magnitude of $E_f = E_i + E_r$, it is obviously best to choose that brush position for which the sparking voltage at the lightest load is equal in value but opposite in sign to the sparking voltage at the highest load. For a certain intermediate load the sparking voltage will then be zero.

This case is represented graphically in fig. 2, in which, for convenience, the negative values of E_i have been plotted against the load. It is seen that

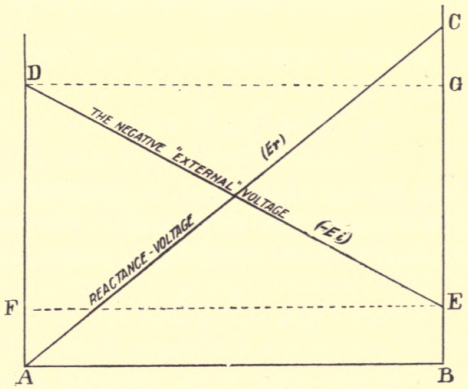

FIG. 2.—The reactance voltage and the external voltage as a function of the load of a shunt motor.

the reactance voltage increases from zero-value at no-load to a certain maximum value at full-load, while the 'external' voltage E_i diminishes from a certain value at no-load to a certain smaller value at full-load. This decrease of E_i is due to the increase of the armature A.T., which oppose more and more the steady field flux linked with the short-circuited coil. E_i is the resultant of the E.M.F. due to the armature flux and the E.M.F. induced by that

part of the field flux which cuts the coil undergoing commutation.

If E_a be the E.M.F. due to the armature flux at full-load, and E_p denote the E.M.F. corresponding to the field flux, we have

at full-load $\quad E_i = E_a + E_p$;

at half-load $\quad E_i = \frac{E_a}{2} + E_p$.

Now, the reactance voltage at half-load is $\frac{E_r}{2}$. Hence, sparkless commutation at half-load obtains when

$$E_p + \frac{E_a}{2} + \frac{E_r}{2} = 0.$$

At no-load the sparking-voltage is equal to E_p since the other voltages are very small. The value of E_p is obtained from the foregoing equation and is

$$E_p = -\frac{E_r + E_a}{2}.$$

This, then, is the value of the sparking voltage at no-load.

Similarly, at full-load, $E_f = E_r + E_a + E_p$, or, by substituting E_p in terms of E_r and E_a,

$$E_f \text{ (at full-load)} = +\frac{E_r + E_a}{2}.$$

Thus, by fixing the brushes in such a position that the voltage E_p induced in the short-circuited coil by the field flux equals $-\frac{E_r + E_a}{2}$, the sparking voltage of the motor will change from a certain positive

value at full-load to the same negative value at no-load. For any other brush position the sparking voltage either at no-load or at full-load would be higher than $\pm\frac{E_r+E_a}{2}$. It is thus seen that the particular brush position as defined above is actually the best.

For reversible motors the brushes must be permanently fixed in the neutral position. In such cases the maximum sparking voltage occurs at full-load, its value being

$$E_f = E_r + E_a.$$

It follows that the maximum sparking voltage of a motor with its brushes in the neutral position is twice that of the same motor whose brushes have been shifted to the best position.

This is not quite correct, however, because both E_r and E_a increase to a certain extent when the brushes are shifted out of the neutral position. This increase is due to a diminution of the reluctance of the magnetic circuit concerned, and it may be considerable if the commutation of the coils takes place near the pole-tips. Professor Arnold has published much information on this point.

Whatever be the brush position chosen, the commutation always depends on E_r+E_a, and it behoves the designer to reduce the value of this expression as much as possible. E_r, the reactance voltage, is generally much larger than E_a, so that it is permissible, as a rule, to use E_r alone as criterion for the commutation of normal dynamos. Cases may occur, however, where the neglect of E_a would lead to a considerable error.

Nearly every dynamo can be cheapened by permitting a higher reactance voltage. There is one exception—namely, motors having a low speed or a low voltage compared with the output, such as, say, a 100 kw. dynamo for 110 volts and 70 revolutions per minute, and so on.

Although, therefore, a reduction in the reactance voltage generally increases the cost of manufacture, there is a large class of machines for which this increase forms but a small percentage of the total costs. In many cases, indeed, an increase in the number of commutator segments—without enlarging the armature—will considerably reduce the reactance voltage; in the case of large dynamos, however, having already only one turn per segment, this means is of no avail, and in such instances the question of reducing the reactance voltage becomes a direct question of cost. To adopt an unnecessary low reactance voltage means a very expensive dynamo; in fact, for a fairly wide range of outputs and speeds it is impossible to reduce the reactance voltage below a certain limit. Such is the case, for instance, with turbo-generators.

From a commercial point of view the reactance voltage has to be fixed as high as is consistent with satisfactory commutation.

So far, we have considered the ordinary type of d.c. machines only, without auxiliary poles or compensating windings—devices which, probably, will be frequently employed in future. It is clear from what has been said above that, in order to keep the sum of reactance and 'external' voltage as small as possible throughout the whole range of output, it

would be desirable to increase the 'external' voltage E_i proportionally with the dynamo load. This may be achieved by providing, in the neutral zones, auxiliary poles whose windings are in series with the armature. The same result may also be obtained by a distributed compensating winding, whose electrical axis is displaced by half the pole-pitch from the axis of the field winding. The A.T. of such a winding (which is also connected in series with the armature) directly oppose the armature A.T. Such a compensating winding was first suggested by Déri, and its effect with regard to the bettering of the commutation is equivalent to the effect obtained with auxiliary poles. In either case the A.T. of the compensating device must be larger than the armature A.T., as only then is a resultant flux obtained which advantageously affects the commutation.

Fig. 3 shows how both the reactance voltage and the 'external' voltage increase with the load of a dynamo fitted with auxiliary poles or a compensating winding. Curve 2 is plotted with the negative values of the external voltage E_i.

If the reluctance of the iron path pertaining to the compensating winding were zero, a complete compensation of the reactance voltage at all loads would be obtainable. But compensating devices are generally used for dynamos only in which the reactance voltage is large; in such cases highly saturated teeth cannot always be avoided. This means that the curve representing E_i is not a straight line but a curve, as indicated in fig. 3. It is then best to adjust the compensating winding so

that between no-load and about three-quarter load the external E.M.F. somewhat over-compensates the reactance voltage, while above three-quarter load E_i does not quite counterbalance E_r. Some firms alter the A.T., as the load changes, by inserting a few resistances in steps. A closer compensation is obtained by this means.

It must be remembered that the reactance voltage

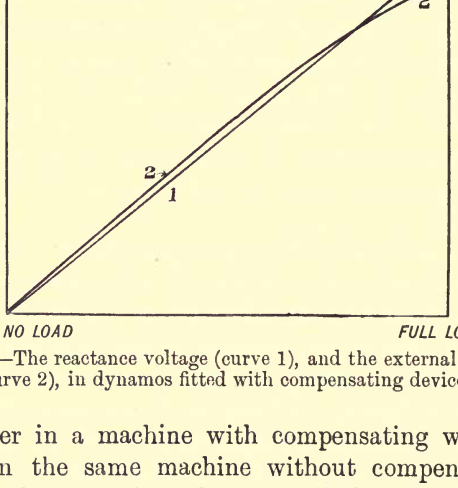

FIG. 3.—The reactance voltage (curve 1), and the external voltage (curve 2), in dynamos fitted with compensating devices.

is higher in a machine with compensating winding than in the same machine without compensation. This is because the reluctance of the path of the flux linked with the short-circuited coils is diminished by the auxiliary poles. The advantages to be gained in the way of commutation by employing compensating devices are nevertheless great; a reactance voltage about 3 to 5 times that of ordinary dynamos may be permitted, and thus it is even possible to

design turbo-generators which commutate satisfactorily. The author holds the opinion, however, that it is a mistake to provide dynamos of moderate outputs and moderate speeds with compensating windings—as is sometimes done—because both the cost and the losses are unnecessarily increased thereby.

Specific Contact Resistance.

It was shown above that the currrent under the brushes automatically distributes itself in such a manner that the voltage $e_2 - e_1$ (fig. 1) attains a definite value, this value depending on the constants of the machine, on the load, and so forth, as explained in the first chapter. It follows that the higher the specific contact resistance between brush and commutator, the smaller need be the difference of current densities at various parts of the brush to produce $e_2 - e_1$. The correctness of this assertion is proved by the superiority of carbon brushes over copper brushes. Carbon brushes themselves differ greatly in contact resistance, so that the permissible reactance voltage depends on the kind of carbon brush employed in any particular case.

It is quite possible that further advances will be made in this direction and that eventually even turbo-generators may be run satisfactorily by this means alone. High-resistance carbon brushes, of course, mean a comparatively large commutator. The cost of the commutator forms, as it is, a large percentage of the total cost of high-speed generators and motors, and this percentage is still further increased by increasing the contact resistance of the brush. If,

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moreover, the voltage is low, it would be a distinct disadvantage to employ high-resistance carbon brushes. For high-tension generators and motors, on the other hand, high-resistance brushes for reducing sparking are promising.

The fact that a number of firms employ copper brushes in lieu of carbon brushes for turbo-generators is due to the large friction losses and 'dancing' of the latter kind of brush. These friction losses mean a larger and costlier commutator, the mechanical construction of which often presents difficulties.

The carbon brush to be used in any particular case must be chosen with due regard to all circumstances. On the one hand, the carbon resistance must be high enough to ensure satisfactory commutation; on the other hand, it must not be unnecessarily high, else extra losses will result. The best specific contact resistance for a given case depends chiefly on the sparking voltage per segment, the number of segments simultaneously covered by a brush and the normal current density under the brush.

The following example[1] will make this clear:—

Total current of motor, . .	620 amperes.
Number of brush sets, . .	8
Number of brushes per set, .	6
Width of brush along the commutator, ...	3·6 cm.
Thickness of brush, measured peripherally, ...	1·2 „
Normal current-density, .	6 amp. per sq. cm.

[1] This example is taken from the calculation of a single-phase series motor in Part II. of this book, where E_f is calculated to be 2·55 volts.

Number of segments . . .	384
Width of segment plus insulation,	0·41 cm.
„ „ without „	0·33 „
Sparking voltage per segment,	2·55 volts.

If we use a brush having a contact resistance of 0·2 ohms per sq. cm., the voltage drop under one brush will be $6 \times 0{\cdot}2 = 1{\cdot}2$ volts, assuming the current to be distributed evenly. But we know from Chapter I. that the current distribution, far from being even, is such that between adjacent segments a voltage equal to the sparking voltage, *i.e.* 2·55 volts, is produced. The difference in potential between the extreme ends of the brush is, therefore, $3 \times 2{\cdot}55 = 7{\cdot}65$ volts.[1] Fig. 4 represents the brush and part of commutator. The voltage between segment and brush at A differs from that at B by 7·65 volts.

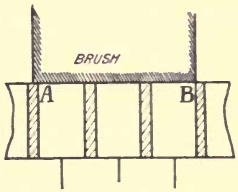

FIG. 4.

The current increases from segment to segment in steps, and the curve representing the distribution of current beneath the brushes ought, for this reason, to be a stepped curve at any given moment. If we plot this stepped curve for every position of the brush relative to the segments, however, it will be at once observed that we are not far from the truth if we represent the *average* distribution of voltage and current by the straight line A′B′ (fig. 5). AB represents the thickness of the brush. Draw line

[1] This voltage is called 'sparking voltage per brush' (E_{sb}), and is dealt with more fully on p. 29.

C E so that A C = 1·2 volts, and draw A′ B′ to intersect C E at a point D, midway between E and C. Make the slope of A′ B′ such that $E B' = C A' = \frac{7 \cdot 65}{2}$.

Obviously, fig. 5 represents the simplest manner of voltage and current distribution to satisfy the conditions outlined above.

B B′ = 3·8 + 1·2 = 5 volts, corresponding to a cur-

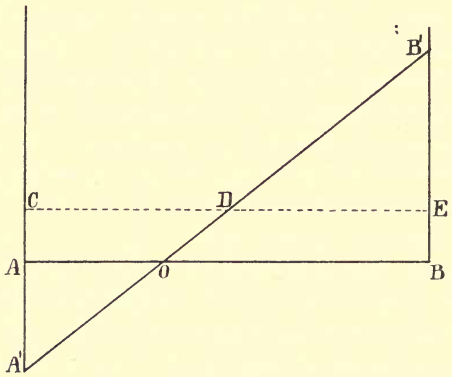

Fig. 5.—Distribution of current under the brushes, the sparking voltage per segment being 2·55 volts and the contact resistance 0·2 ohms per sq. cm.

rent density at B of 5/0·2 = 25 amperes per sq. cm. Further, A A′ = − 3·8 + 1·2 = − 2·6 volts, corresponding to a current density at A of 13 amperes per sq. cm.

It will be gathered from fig. 5 that the current at A flows in the opposite direction to that at B. The actual distribution is somewhat better than indicated in fig. 5, not only because part of the sparking voltage is absorbed in the armature winding and

commutator leads, but also because the distribution does not follow—as has been already remarked—a straight-line law. Its real shape is a wavy curve, as has been proved experimentally by Professor Arnold. By assuming this straight-line law, however, we are on the safe side.

Let e_v = voltage drop under a set of brushes for an evenly distributed current (e_v = A C in fig. 5), and e_u = effective drop of voltage under a set of brushes for an unequal distribution of current

then, $$e_u = e_v + \frac{E_{fb}^2}{12 e_v} \quad . \quad . \quad \text{(III.)}^1.$$

This effective voltage drop, e_u, when multiplied by the motor current, gives the C^2R losses under the brushes at imperfect commutation.[2]

In our example $e_v = 1{\cdot}2$ volts and $E_{fb} = 7{\cdot}65$ volts;

hence $$e_u = 1{\cdot}2 + \frac{7{\cdot}65^2}{12 \times 1{\cdot}2} = 5{\cdot}2 \text{ volts};$$

Thus we arrive at the surprising result that the C^2R losses under the brushes are increased more than fourfold compared with the losses at perfect commutation.

[1] This equation, expressed in words, means that the sparking voltage leads to an increase in the commutator losses of the amount

$$= \frac{E_{fb}^2}{3 \times \text{contact resistance}}.$$

[2] Proof: Let E be the contact E.M.F. at any point. In fig. 5 take a point on A B vertically under D as origin. Then $E = E_{fb} x + e_v$.

C^2R loss at any point $= \frac{E^2}{r}$; total C^2R loss $= \int_{-\frac{1}{2}}^{+\frac{1}{2}} \frac{E^2}{r} dx = \frac{E_{fb}^2}{12r} + \frac{e_v^2}{r} =$

$$\frac{e_v}{r}\left(e_v + \frac{E_{fb}^2}{12 e_v}\right) = \text{motor current} \times \left(e_v + \frac{E_{fb}^2}{12 e_v}\right).$$

If, instead of using brushes with a contact resistance of 0·2 ohms per sq. cm., harder brushes—having, say, a contact resistance of 0·3 ohms per sq. cm.—had been chosen, the current distribution in our example would have been as indicated in fig. 6, in which

$$A\,C = 6 \times 0{\cdot}3 = 1{\cdot}8 \text{ volts, } E\,B' = 3{\cdot}8 \text{ volts and } A\,A' = -2{\cdot}0 \text{ volts}$$

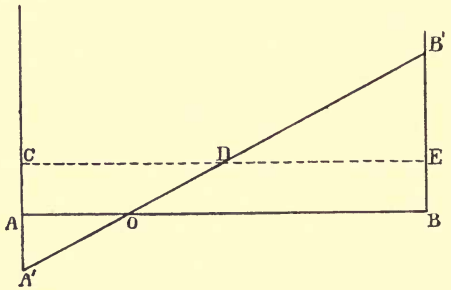

Fig. 6.—Distribution of current under the brushes, the sparking voltage per segment being 2·55 volts and the contact resistance 0·3 ohms per sq. cm.

The effective voltage drop under a brush would then come to

$$e_u = 1{\cdot}8 + \frac{7{\cdot}65^2}{12 \times 1{\cdot}8} = 4{\cdot}4 \text{ volts.}$$

For a brush with the still higher contact resistance of 0·4 ohms per sq. cm., we get

$$e_u = 2{\cdot}4 + \frac{7{\cdot}65^2}{12 \times 2{\cdot}4} = 4{\cdot}4 \text{ volts.}$$

The C^2R losses have thus not been reduced any further by using this high-resistance carbon, and if the contact resistance be increased beyond 0·4 ohms per sq. cm. the losses will increase again. It is seen,

therefore, that there is one certain contact resistance in each particular case which will give a minimum C^2R loss under the brushes. The following table contains the chief results for brush-contact resistances of 0·2, 0·3, 0·4, and 0·5 ohms per sq. cm. applied to our example:—

TABLE I.

Contact resistance per sq. cm.	0·2	0·3	0·4	0·5
Voltage at point A, . .	− 2·6	− 2·0	− 1·4	− 0·8
,, ,, B, . .	5·0	5·6	6·2	6·8
Current-density at point A,	−13	− 6·7	− 3·5	− 1·6
,, ,, B,	+25	+18·7	+15·5	+13·6
Effective drop of voltage under a brush, . .	5·2	4·4	4·4	4·6
Effective drop of voltage under positive and negative brushes, .	10·4	8·8	8·8	9·2
C^2R losses under the brushes (in watts), .	6400	5400	5400	5700
Loss at point B per sq. cm. (volts × current), .	125	105	95	92

The choice of brush in our example lies between one having a contact resistance of 0·3 ohms per sq. cm. and one having a contact resistance of 0·4 ohms per sq. cm. The former brush is better, however, since, generally speaking, that part of the sparking voltage which is lost in the short-circuited coils is larger the smaller the ratio of contact resistance to the resistance of the short-circuited coils and commutator leads. With a contact resistance of 0·3 ohms per sq. cm., therefore, the sparking voltage at the brush will be somewhat smaller than with a contact resistance of 0·4 ohms per sq. cm.

e_u is, obviously, a minimum for $e_v = \frac{E_{fb}}{2\sqrt{3}}$; or, in our example, when $e_v = \frac{7{\cdot}65}{2\sqrt{3}} = 2{\cdot}2$ volts. This corresponds to a contact resistance of $2{\cdot}2/6 = 0{\cdot}37$ ohms per sq. cm. For a brush with such a contact resistance the C^2R losses become a minimum.

It is noteworthy that the effective drop of voltage may be larger than the maximum drop of voltage between brush and commutator (see Table I.). This arises from the fact that part of the current flowing from brush to commutator is opposite in direction to the remaining part of the current. Thus the C^2R losses and the effective drop in voltage are greatly increased, while the maximum voltage drop does not increase in the same proportion.

Manufacturers generally state the resistance per cubic cm. of their brushes, but often omit to mention the contact resistance. But these two resistances are obviously not identical and both ought always to be given.

Experiments showing that a high contact resistance improves commutation were first made known by Mr Dunn in the discussion on Mr Reid's paper read before the American Institute of Electrical Engineers.[1] Mr Dunn suggested that the brush be made of a mixture of carbon and clay in varying proportions so as to obtain contact resistances of any value desired.

The preceding calculations were based on the assumption that the contact resistance is the same at all points of the brush whatever value the

[1] *Transactions of the American Institute of Electrical Engineers*, vol. xiv., 1897, p. 591.

current density may assume. It has been averred by others, however, that this contact resistance is dependent on the current density.

As a result of experiments, Herr Kahn and others even go so far as to suggest that the *voltage drop* under the brush is constant whatever the current density employed. Accordingly, the drop in voltage should be the same, whether, say, the current density were two amperes per sq. cm. or seven amperes per sq. cm. If this were true, the voltage drop would be the same at every point of the contact area between brush and commutator. But it is easily proved experimentally that this is not the case. The author does not think that the contact resistance is directly affected by the current density, but he believes that it might be influenced indirectly by the negative temperature coefficient of carbon. Herr Kahn's experiments were made with slip-rings, and the results obtained with them cannot be utilised immediately in connection with commutators. With slip-rings the current density is approximately the same at all points of the contact area, and the temperature of the brush is chiefly dependent on the current density at the time being.

In the case of a brush bearing on a commutator, however, the temperature at a given point is not only dependent on the current density at the point considered but also on the current density in other parts of the brush.[1] This is because the heat is

[1] Since the appearance of the German edition of this book, this point has been independently investigated by Mr Reid. (See the *Transactions of the American Institute of Electrical Engineers*, vol. xxiv, p. 297.)

conducted from the hotter parts of the brush to the colder points.

Nevertheless, there still remains a difference in temperature between the various parts of the brush, and this difference affects the commutation adversely. In figs. 5 and 6, for instance, the temperature at B will be higher than at point A, and the resistance will consequently be smaller at B than at A. This means that the distribution of current under the brush must be even more unequal than shown in the two diagrams in order that the given sparking voltage be produced. This increase of current density at B gives rise to a still higher temperature at this point, and this again has the effect of further reducing the resistance at B, and so on until a final state is reached. For this reason it would be of advantage to possess brushes having a high contact resistance and a positive temperature coefficient.

Resistances in the Commutator Leads.

It has been proposed to assist the resistance between brush and commutator by inserting resistances between the armature winding and the commutator segments. Opinions differ as to the value of this expedient. Mr Mordey, for instance, is not in favour of this means; others, however, claim to have obtained satisfactory results in this direction, and at present such resistances are often suggested in connection with single-phase motors.

The total resistance is made up of the contact resistance between brush and commutator and the resistance in the commutator leads. Both are in

series. But the contact resistance between the brush and a segment increases automatically as the segment leaves the brush, while the resistance in the commutator lead remains constant. When the brush covers the whole, $\frac{3}{4}$, $\frac{1}{2}$ and $\frac{1}{4}$ segment, the corresponding brush contact resistances are proportional to 1, $\frac{4}{3}$, 2 and 4 respectively. The resistance in the commutator lead forms a variable percentage of the brush contact resistance, and as the segment leaves the brush this percentage grows smaller and smaller until it reaches zero-value; this occurs at the moment the brush breaks contact with the segment.

The *specific* contact resistance of the brush remains, of course, the same whether the brush covers a whole segment or only part of it; but the additional specific resistance due to the resistance in the commutator lead varies from zero (when the brush touches a segment) to a certain maximum value (when the brush covers a whole segment). This additional specific resistance is numerically equal to (resistance in commutator lead) × (contact area between brush and segment).

In fig. 7, the line B G represents the constant specific contact resistance, while B E F G represents the additional resistance per sq. cm. of contact area accruing from the auxiliary resistance in the commutator leads. The minimum resistance per sq. cm. (represented by A–B) occurs when the brush is just making contact with the segment considered; as the segment moves under the brush, the specific total resistance increases to a maximum C E, this maximum being maintained as long as the segment

remains totally covered by the brush. At H, the segment emerges from under the brush and the total resistance per sq. cm. of contact area begins to decrease. At G, the segment is on the point of leaving the brush.

Thus the following conclusion is arrived at:—The resistance in the commutator lead exerts its full influence as long as the respective segment is fully covered by the brush. When the time (represented by C–H in fig. 7) during which the segment is wholly covered by the brush is long compared

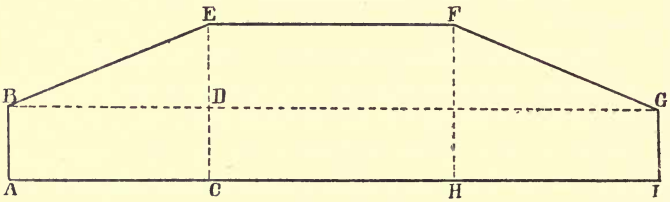

FIG. 7.—Diagram showing the variation of the specific contact resistance with the time. A B G I specific contact resistance of the carbon brush, B E F G additional specific contact resistance corresponding to the resistance in the commutator lead.

with the time (A–C and H–I) during which the segment is being covered (or uncovered) by the brush, or, in other words, when the brush covers many segments, the resistance in the commutator lead is nearly as effective as the same resistance in the form of contact resistance proper. Conversely, resistance in the commutator leads is much less effective than an equal amount of contact resistance in cases in which the brush only covers one or two segments. In such conditions detrimental pulsations of current will result, varying from an amount pro-

portional to A–B to a value proportional to C–E. The smaller the contact resistance in comparison with the total resistance (*i.e.* the contact resistance plus the auxiliary resistance in the commutator lead), the more pronounced these pulsations.

The simple diagrams representing the current distribution (figs. 5 and 6) were constructed on the assumption that the specific contact resistance was always the same at every point of the brush contact. In the more complicated case of fig. 7, the specific contact resistance varies, as explained, with the time. This case must be dealt with by assuming an average specific resistance, and by considering separately the influence of the fluctuations in the current density.

The foregoing reflections lead to the following rule :—Do not use resistance in the commutator leads as long as satisfactory commutation can be secured by using ordinary brushes. But if the reactance voltage cannot be dealt with any longer by carbon brushes alone, the specific contact resistance may be supplemented by inserting resistances in the commutator leads. The larger the number of segments simultaneously covered by one brush, the greater the effect of these resistances.

Thickness of Brush.

Let n denote the number of segments under one brush, and E_f, as before, the difference of the voltage drop between a segment and the brush and the adjacent segment and the brush. Then the sparking voltage per brush (E_{fb}) is

$$E_{fb} = nE_f.$$

In our example (p. 19), for instance, $n = 3$ and $E_f = 2{\cdot}55$ volts. Hence $E_{fb} = 3 \times 2{\cdot}55 = 7{\cdot}65$ volts. This equation, however, is only correct when the brush covers many segments. If the brush is only as wide as one segment, for example, the brush will make contact at all times with two segments only, and the current-density in the contact area of each segment is merely

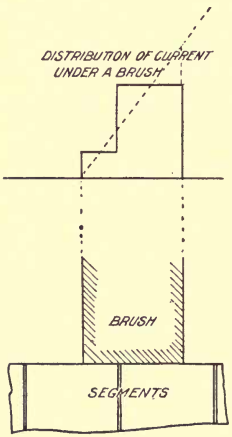

FIG. 8.—Distribution of current under a brush the width of which is equal to the width of one segment.

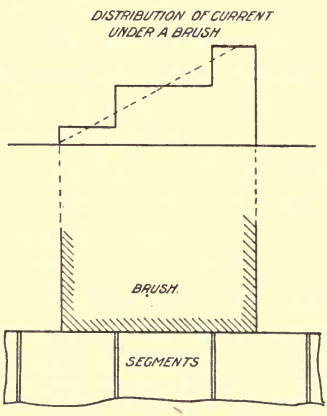

FIG. 9.—Distribution of current under a brush covering two segments.

a function of the time. At a given moment, the distribution of current will be as shown in fig. 8. Fig. 9 represents the distribution—at a given moment—in the case of two segments being covered by the brush. It is certain that in these two cases the current fluctuates considerably with the time, and it is evident that the fluctuations are smaller the greater the number of segments simultaneously covered by one brush.

These fluctuations were dealt with by the author in an article entitled "Zur Theorie der Stromwendung" which appeared in the *Zeitschrift für Elektrotechnik* (1902, No. 30). The factor a, mentioned in that article, has the following meaning:

$$a = \frac{\text{Sparking voltage per segment}}{\frac{1}{2}\ \text{Sparking voltage per brush}}.$$

It was shown that a had the following values:

$a = 1$ for a brush-width = width of segment + insul.

$a = \frac{3}{4}$ " " $= 2 \times$ { width of one segment + insulation.

$a = \frac{5}{9}$ " " $= 3 \times$ " "

$a = \frac{7}{16}$ " " $= 4 \times$ " "

$a = \frac{2n-1}{n^2}$ " " $= n \times$ " "

By calculating the sparking voltage per brush with the aid of a ($E_{fb} = \frac{2}{a} E_f$), one obtains directly the worst distribution of current under the brush. For $\frac{2}{a}$ we may write

$$a' = \frac{2}{a} = \frac{2n^2}{2n-1} = \frac{n}{1 - \frac{1}{2n}}.$$

The expression $\frac{1}{1 - \frac{1}{2n}}$ indicates at once the fluctuations of current density at any point of the contact area. If the brush covers many segments, *i.e.* if n be large, the expression $\frac{1}{1 - \frac{1}{2n}}$ is very nearly unity and

we may write, as was done above,

$$E_{fb} = n \times E_f.$$

Thus the following conclusion is arrived at:—

The sparking voltage per brush increases with the number of segments covered by the brush.

It is possible to improve the commutation of any dynamo by reducing the peripheral thickness of its brushes, provided that the normal current density is not greatly exceeded thereby. This means may be employed to its full extent by lengthening the commutator as the brush is reduced in thickness, so that the contact area remains the same. This lengthening of the commutator implies, of course, increased cost, and it must also be borne in mind that with thin brushes the expression $1 - \frac{1}{2n}$ is no longer nearly unity.

It is wrong, in any case, to employ a brush which is thinner than one segment; indeed, the author is of the opinion that the brush ought not to be less than $1\frac{1}{2}$ times as thick as a segment. Frequently, however, a thicker brush is necessary for mechanical reasons.

The formula given on p. 7 for the calculation of the reactance voltage is based on the assumption—suggested by Hobart—that each ampere-turn of the armature winding produces 4 lines of force per cm. of 'embedded' length and 0·8 lines of force per cm. 'free' length. These values represent the averages of many observations, and they take particularly into account that a large portion of the lines of force created give rise to mutual-induction effects as well as to self-

induction effects. Obviously, the mutual-induction forms a larger percentage of the total induction if the number of segments covered by a brush is large than if this number is small. This means that smaller figures than 4 and 0·8 are permissible when the brush covers many segments, and that larger figures ought to be taken when only a few segments are covered by a brush.

From the foregoing we may conclude that too great a reduction of the peripheral thickness of the brush is next to useless in bettering the commutation, since, as has been indicated, the mutual-induction forms a smaller and smaller percentage of the total induction as the number of segments under a brush diminishes.

If two equal coils are in the same slot, the lines emanating from coil 1 will produce in coil 2 an E.M.F. of mutual-induction which is nearly the same as the E.M.F. of self-induction produced by the same lines in coil 1. In this case, therefore, the ratio of mutual-induction to self-induction is practically unity. But if the two coils are embedded in neighbouring slots, this ratio (denoted by μ) lies between 0·4 and 0·6 and assumes even smaller values if the coils are still farther apart; μ is also largely dependent on the pole-pieces, whether these are solid or laminated. It is difficult to calculate μ with any degree of accuracy, but it may easily be determined experimentally. The average value of μ for all the coils short-circuited by the brushes will, as a rule, be between 0·4 and 0·8.

Assuming no E.M.F. to be induced in the short-circuited coil from outside, the sparking voltage per

brush is represented in fig. 10 as a function of the number of segments under the brush. This figure is reproduced from the author's article cited above. The curves are plotted for $\mu = 1$, $\mu = 0{\cdot}75$, $\mu = 0{\cdot}5$ and $\mu = 0{\cdot}25$. The ordinates represent relative values of the sparking voltage per brush, and the

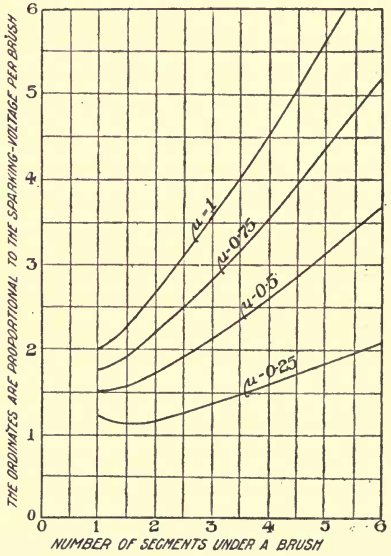

FIG. 10.

curves show that the gain in the way of bettering the commutation by reducing the peripheral thickness of the brush is greatest for $\mu = 1$. It is seen, on the other hand, that the tendency to spark may be rendered even greater by making the brush thickness less than $1\frac{1}{2}$ times that of the segment in cases where μ has as low a value as 0·25.

It is very important to choose the proper number of commutator segments. With small motors for medium and high voltages, the armature turns per segment can be reduced in the same degree as the number of segments is increased. But if the same brush is used as before, the sparking voltage per brush will not have diminished in the same proportion. Take as example the case in which the brush covers 3 segments. If now the number of segments be doubled, there will be 6 segments beneath the brush. Although the sparking voltage per segment has been reduced to half its former value, the sparking voltage per brush has probably not fallen below 85 or 90 per cent. of its original value. But if the brush is reduced to half its former thickness at the same time, and twice the original number of brushes are provided so as to maintain the original current density under the brush, then the sparking voltage per brush will have been reduced to about 55 or 60 per cent. of its first value. It will not have fallen to quite half its former value because the factor μ has increased somewhat as a result of the change. This increase of μ is due to the fact that the coils which are simultaneously short-circuited now lie nearer together than before.

Instead of reducing the brush in thickness, we might have left it as before and inserted resistances in the commutator leads.

The commutation is thus always improved by increasing the number of segments, and the question remains at what price this is obtained. If the gain in this direction were, say, only 10 per cent. as a

result of doubling the number of segments, it would be worth while to consider whether the same result might not be arrived at cheaper by re-designing the armature. This can, of course, only be decided from case to case. It may be remarked, however, that it is almost always advisable to double the number of segments if the brush only covers 1 or $1\frac{1}{2}$ segments in the first instance. It is then best to retain the brush, and the sparking voltage per brush will be reduced by from 25 to 40 per cent. approximately. But if the dynamo has only one turn per segment, the doubling of the segments is of no avail unless the number of poles or the armature A.T. per pole are altered at the same time.

CHAPTER IV.

WHAT VALUES OF THE REACTANCE VOLTAGE, SPARKING VOLTAGE PER SEGMENT, AND SPARKING VOLTAGE PER BRUSH ARE PERMISSIBLE?

It lies in the nature of the phenomenon of sparking that a direct and general answer to this question cannot be given. Very often certain values of reactance voltages are mentioned as permissible without qualification of the conditions to which these reactance voltages refer. It is frequently not stated, for instance, what kind of carbon brushes were used, or the number of segments covered by one brush, or the E.M.F. induced in the short-circuited coils by the armature flux, or whether the brushes were in the neutral position or not. Another important point

to be considered in this connection relates to the kind of armature winding employed. All other things equal, the reactance voltage allowable is higher for lap-wound armatures and lower for wave-wound armatures. Experiments in this direction have shown that a 4-pole machine with lap-wound armature, which began to spark at a reactance voltage of 3 volts, commenced to spark already at from 1·8 to 2·2 volts when provided with a wave-winding. This is a case which illustrates that the reactance voltage is not nearly so good a guide as to sparking as is the sparking voltage per brush. It can be shown, in fact, that though the reactance voltage may be the same in the case of both windings, the sparking voltage per brush is considerably larger in the case of the wave winding.

In small motors, in which a lap winding would necessitate two or more turns per segment, it is, however, of advantage to employ a wave winding since the reactance voltage will be reduced thereby. In a 4-pole motor, for example, the reactance voltage can be reduced to half by substituting a wave winding for a lap winding.

In stationary motors, the number of brush-sets ought always to be equal to the number of poles, because, for a given commutator, the thickness of the individual brush is inversely proportional to the number of brush-sets. It is the author's opinion that this is the chief advantage of having many sets of brushes. If two of the four brushes of a 4-pole motor with wave winding be lifted off the commutator, it is observed, as a rule, that the sparking increases. But such a comparison is deceptive,

inasmuch as the current density under the brush is doubled.

The influence of all details of design on the commutation may be computed with some degree of accuracy, but the quality of the brushes used cannot yet be expressed in figures. True, the contact resistance can be measured, but this changes with the temperature. For one and the same quality of brush, however, any dynamo designer may easily and speedily determine experimentally all those constants which will enable him to predict with sufficient accuracy the commutation of any dynamo.

CHAPTER V.

THE COMMUTATION OF SINGLE-PHASE COMMUTATOR MOTORS.

THE problem is much simplified by considering motors only in which the frequency of the alternating supply is low compared with the frequency of commutation. By frequency of commutation is meant 2/T, T being the time, in seconds, during which a coil is short-circuited. In such cases the armature current remains practically constant during the time the armature coil in question undergoes commutation. The reactance voltage in the armature coil which happens to be under the brush at any particular moment is proportional to the line current at the time being, and it is clear that the momentary reactance voltages induced in successive coils as they pass quickly, one by one, under the brushes, grow

and fall with the line current. We may say, therefore, that the reactance voltage of the series motor as a whole (as distinct from the reactance voltage of an individual coil) is in phase with the line current. The E.M.F.'s which then appear in the short-circuited coil are the same as obtain in continuous-current motors, viz :—

(1) The reactance voltage E_r;

(2) The E.M.F. E_i induced from outside; and

(3) The E.M.F. E_f due to the unequal distribution of current under the brushes.

Obviously, we have as before:

$$E_f = E_i + E_r.$$

This equation holds good for every instantaneous value of the current, and if both E_i and E_r follow the sine law, the phase and value of E_f is obtained by adding E_i and E_r vectorally.

Consequently, in the case of a single-phase motor, complete compensation (i.e. $E_f = 0$) *is only obtained if E_i and E_r have the same magnitudes but are displaced in phase by* 180 *degrees.*

If E_i and E_r do not follow the sine law, as is very often the case, complete compensation can only be obtained if a third condition is also fulfilled, this third condition being that both E_i and E_r follow the same curve.

There are a few motors for which a complete compensation is theoretically possible, and if—in such cases—high values of E_i and E_r are adopted, or if copper brushes are to be employed, great care must be taken to ensure that the curves representing the change of E_i and E_r with the time are

identical; otherwise the commutation may be far from perfect.

For the three well-known types of single-phase motor (viz. the series motor, repulsion motor, and compensated repulsion motor) a complete compensation of the sparking voltage at all speeds is quite impossible, and the condition that the curves of E_i and E_r be identical loses its importance since the only way to prevent sparking in these cases is to reduce E_i and E_r to such values that their resultant E_f is not large enough to cause trouble. The reactance voltage E_r is always in phase with the supply current, since the frequency of supply is very low compared with the frequency of commutation; but the "external" voltage E_i may have a lead or a lag relative to the wave of the supply current according to the type of motor and the latter's load at the time being.

Since E_r, E_i, and E_f are alternating, the question will have to be discussed whether their maximum values or their effective values have to be considered. As a rule, it will be correct to take the effective values; this is even correct when comparing a d.c. machine with a single-phase motor as regards sparking. There is one exception, however—viz. when the motor rotates at synchronous speed or at a multiple of the synchronous speed for a long time. In this case it is correct to take the maximum value of E_f into account; this will be evident when it is remembered that at such speeds the maximum value of E_f always occurs when one and the same segment is under the brush. The wear of this segment would then depend on the maximum value of E_f. But such cases are so infrequent that they need

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hardly be considered, at least not in connection with motors for ordinary purposes.

The formula for the effective reactance voltage of single-phase motors can be deduced from the formula quoted on p. 7 for the reactance voltage of d.c. dynamos. It is clear that with the same armature, the same current and at the same speed, the effective reactance voltage will be the same for alternating current as for continuous current. In the case of the d.c. dynamo, the counter-E.M.F. produced in the armature winding is

$$E = 4 \times N' \times s \times M \times 10^{-8} \text{ volts},$$

in which

N' = number of pole-pairs × revs. per second;
s = armature turns between the brushes; and
M = flux from one pole.

In the alternating-current motor, on the other hand, the effective value of the counter-E.M.F. produced by rotation is

$$E = \frac{4}{\sqrt{2}} N' \times s \times M \times 10^{-8},$$

in which M = maximum flux from one pole.

We obtain, therefore, the reactance voltage of the single-phase motor by substituting in the formula for the reactance voltage in d.c. dynamos $\frac{M}{\sqrt{2}}$ for M and by introducing for the input A the product $J \times E$, J being the effective current.

Hence for single-phase motors

$$E_r = \frac{4\sqrt{2}\, z\, J\, E(g + 0{\cdot}1g')}{pM} \quad . \quad . \quad \text{(IV.)}.$$

This formula is based on the same number of lines of force per cm. winding as were adopted in connection with the d.c. dynamo. But it is probable that a higher figure for the lines of force per cm. embedded length of winding ought to be taken if the motor has a distributed field winding like an induction motor and not well-defined pole-pieces such as are used for d.c. dynamos. Tests on single-phase motors of the former type are scarce, and no experiments with regard to the reactance voltage of such motors have yet been published.

CHAPTER VI.

THE REACTION OF THE SHORT-CIRCUITED COILS ON THE FIELD.

It is generally supposed that a continuous-current armature has no back A.T. when the brushes are in the neutral position; but it was observed by Mr Hobart and the author[1] during some experiments that in certain conditions the short-circuited coils may react on the field flux in the same manner as do back A.T. Such an effect has been observed by Steinmetz[2] in connection with repulsion motors. This author, however, does not mention how great

[1] Since the appearance of the German edition of this book, a treatise by Dr R. Pohl has been published, dealing with this subject of reaction in a very exhaustive manner. Dr Pohl's book is entitled *Über magnetische Wirkungen der Kurzschlussströme in Gleichstromankern.* (Stuttgart: F. Enke. 1·20 marks.)

[2] *Transactions of the American Institute of Electrical Engineers*, vol. xxi. pp. 61–126.

this influence is, nor does he quote particulars which would enable it to be pre-determined. The phenomenon observed is put down by Steinmetz to the transformer action of the short-circuited coils, but it is apparent that the reactance voltage has also something to do with it.

In fig. 11, J denotes the current flowing in one

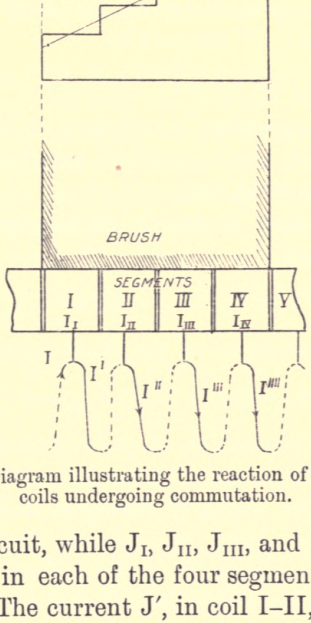

FIG. 11.—Diagram illustrating the reaction of the armature coils undergoing commutation.

armature circuit, while J_I, J_{II}, J_{III}, and J_{IV} represent the currents in each of the four segments covered by the brush. The current J′, in coil I–II, is

$$J' = J - J_I.$$

The current J″ in the armature coil II–III is

$$J'' = J' - J_{II} = J - (J_I + J_{II}).$$

Similarly, the current J''' in the coil III–IV is

$$J''' = J'' - J_{III} = J - (J_I + J_{II} + J_{III}),$$

and the current in the coil IV–V is

$$J'''' = J''' - J_{IV} = J - (J_I + J_{II} + J_{III} + J_{IV}).$$

But, obviously,

$$J'''' = -J \quad \text{and} \quad J_I + J_{II} + J_{III} + J_{IV} = 2J.$$

Assuming one armature turn per commutator segment, the ampere-turns of the three coils I–II, II–III, and III–IV amount to

$$J - J_1 + J - (J_I + J_{II}) + J - (J_I + J_{II} + J_{III}) = 3J - (3J_I + 2J_{II} + J_{III}$$

This simple expression is sufficient to show in what conditions the short-circuited armature coils will react upon the field flux.

If the current is evenly distributed under the brushes, we get

$$J_I = J_{II} = J_{III} = 2J/4 = J/2,$$

and the ampere-turns of the short-circuited coils are

$$3J - (3J_I + 2J_{II} + J_{III}) = 0\,;$$

i.e. *there is no reaction on the field by the short-circuited coils if the distribution of current under the brushes is even, or, in other words, if the commutation is perfect. If, on the other hand, the currents through segments I. and II. are greater than the currents through III. and IV., the above expression assumes a negative value*; *and if the currents in I. and II. are smaller than the currents through III. and IV., the expression becomes positive.*

It is not possible to calculate this reaction quite

accurately, but the following approximate method of calculation suffices for practical purposes.

Introducing $2J = J_I + J_{II} + J_{III} + J_{IV}$ into the expression above, we get

$$\text{Reaction} = \frac{3}{2}J_I + \frac{3}{2}J_{II} + \frac{3}{2}J_{III} + \frac{3}{2}J_{IV} - 3J_I - 2J_{II} - J_{III}$$

$$= -\frac{3}{2}J_I - \frac{1}{2}J_{II} + \frac{1}{2}J_{III} + \frac{3}{2}J_{IV}$$

$$= \frac{3}{2}(J_{IV} - J_I) + \frac{1}{2}(J_{III} - J_{II}).$$

Now, $J_{III} - J_{II}$ can be determined if the sparking voltage per segment and the contact resistance (W) of all the brushes of one set are known. This may be proved as follows:—

It was shown previously that E_f = specific contact resistance × difference of current density in neighbouring segments = W × number of segments × difference of current density in adjoining segments. In our case, therefore,

$$E_f = W \times 4 \times (J_{III} - J_{II});$$

hence

$$J_{III} - J_{II} = \frac{E_f}{4W}.$$

Assuming the current to increase in equal steps from segment to segment, we get

$$J_{IV} - J_I = 3(J_{III} - J_{II}).$$

The reaction of the short-circuited coils is thus in our case

$$\frac{9}{2}(J_{III} - J_{II}) + \frac{1}{2}(J_{III} - J_{II}) = \frac{5E_f}{4W} = 1{\cdot}25\frac{E_f}{W}.$$

This equation holds good in the case in which the

brush covers exactly the four segments, but during the greater part of the time the brush is in contact with five segments. The reaction on the field is then somewhat greater, and a calculation shows that the average value of the reaction is 8 per cent. larger than that quoted. Hence

$$\text{Average reaction on the field} = 1{\cdot}35\,\frac{E_f}{W}.$$

A general formula for the reaction of the short-circuited coils on the field is

$$\text{Reaction} = S \times \frac{\text{Sparking voltage per segment} \times \text{turns per segment}}{\text{Con.-resist. betw. a set of brushes of the commutator}}$$

S being a constant which varies—as seen from the following table—with the number of segments covered by one brush.

S = 0·33	in the case that the brush covers			2	segments.
= 0·78	,,	,,	,,	3	,,
= 1·35	,,	,,	,,	4	,,
= 2·1	,,	,,	,,	5	,,
= 3·0	,,	,,	,,	6	,,
$= 0{\cdot}083n^2$	,,	,,	,,	n	,,

If the general value of S be substituted in the above formula, we get

$$\text{Reaction} = 0{\cdot}083n^2\frac{E_f z}{W} \quad . \quad . \quad . \quad \text{(V.)}.$$

Consider, as an example, a motor having the following particulars: Sparking voltage per segment = 2·5 volts; contact resistance between one set of brushes and the commutator = 0·006 ohms; number of segments simultaneously covered by a brush = 3·5; number of turns per segment (z) = 1.

The reaction of the short-circuited coils upon the flux of each pole is

$$= 0{\cdot}083 \times 3{\cdot}5^2 \times 2{\cdot}5/0{\cdot}006 = 420 \text{ ampere-turns.}$$

It is thus evident that this reaction is not always negligible. Regarding its tendency, the following rule applies:—

If the sparking voltage per segment (i.e. $E_s = E_r + E_i$) *is positive, the ampere-turns of the short-circuited coils increase the field flux of motors and decrease the field of generators; and vice versâ.*

The above considerations and formulæ apply to both d.c. and single-phase machines. The M.M.F. of the short-circuited coils is in phase with the sparking voltage. Since the sparking voltage of single-phase motors is generally out of phase with the field flux, the latter is not only strengthened or weakened, as the case may be, but is also shifted in phase. This effect will be dealt with later on when the different types of single-phase motor are considered seriatim.

The reaction of the short-circuited coils could be made use of in various ways. It would be possible, for instance, to design a reversible d.c. motor without any field winding. The brushes would be fixed in the neutral position. Such a motor would have no starting torque, but when once brought up to a certain speed it would continue to run on its own account. In designing such a motor, care would have to be taken that the ampere-turns of the short-circuited coils are sufficient to create the necessary flux. A motor of this kind would operate even better with single-phase currents, because in

this case the cross A.T. could be counterbalanced by a suitable compensating winding. The air-gap could then be made small even if a large number of armature A.T. per pole were chosen.

Other applications suggest themselves, but the author does not think that the principle will ever be made use of for the reason that the same factors which help to increase the reaction also increase the tendency to spark.

CHAPTER VII.

THE CHANGE OF THE SPARKING VOLTAGE OF CONTINUOUS-CURRENT MACHINES WITH THE LOAD.

Shunt Motors.—Fig. 2 represents the sparking voltage of shunt motors as a function of the load. The reactance voltage E_r (= B C in fig. 2) is calculated by means of the formula given on p. 7. E_i, the 'external' voltage, consists of two components—viz. a component E_a due to the armature flux (flux produced by the armature–A.T.), and another component E_p due to that part of the pole flux which links with the short-circuited coils. E_p varies in value, as a rule, between 0·15 and 0·4 of the average voltage per segment, according to the distance between the pole-tips of adjacent poles, the extent to which the brushes are shifted out of their neutral position, the radial length of air-gap, and the degree of saturation in the armature teeth.

Figs. 2 and 13 are plotted for the best position of

the brush, *i.e.* for $E_p = -\frac{1}{2}(E_r + E_a)$. If the brushes are in the neutral position, however, the various voltages in the coil change with the load as indicated in fig. 12. As a rule, the reactance voltage is considerably larger than the 'external' voltage; but occasionally the reverse occurs.

Series Motors.—The curve representing the increase

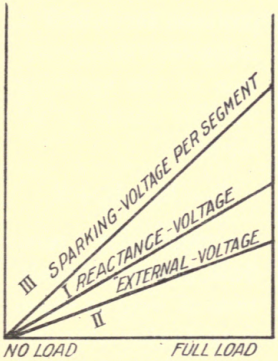

FIG. 12.—Diagram indicating the increase of the different voltages in the short-circuited coils of a shunt motor with the load, the brushes being fixed in the neutral position.

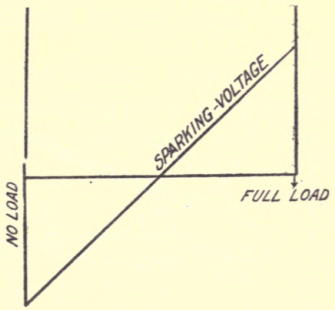

FIG. 13.—Diagram indicating the variation of the sparking voltage of a shunt motor with the load, the motor brushes being fixed in the best position.

in reactance voltage of a series motor with the load depends largely on the prevailing conditions as to saturation. Let curve I. (fig. 14) represent the increase of the flux per pole with the armature current, and curve II. the speed of the motor. The reactance voltage and the voltage E_a due to the armature flux are proportional to current × speed. The curves III., IV., and V. represent the reactance

voltage, the 'external' voltage, and the sparking voltage against the current or the load, the brushes being permanently in the neutral position.

If the brushes now be shifted backwards, another E.M.F. (E_p) appears on account of part of the field flux becoming linked with the short-circuited coils. This E.M.F. opposes both E_r and E_a and is proportional to flux density × speed, or current × speed. Hence there is a certain brush position for which a complete compensation of the sparking voltage occurs at all loads, *i.e.*

$$E_f = E_a + E_p + E_r$$

is constantly zero. This, however, is only true as long as there is proportionality between the current and the field flux which links with the coils. If, on the other hand, the field magnet system is highly saturated, the compensation will be incomplete, yet much better than in shunt motors.

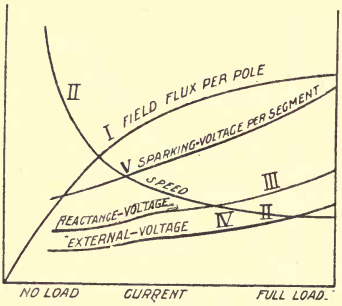

FIG. 14.—Diagram indicating the increase of the different voltages in the short-circuited coils of series motors with the load, the brushes being in the neutral position.

Since series motors are required, as a rule, to run in either direction, their brushes have to be fixed in the neutral position and little use can be made of the above excellent properties.

Compound Motors.—The curve representing the variation of the sparking voltage of compound

motors, the brushes of which have been shifted to the best position, lies between the corresponding curves of the shunt and series motors.

CHAPTER VIII.

THE SPARKING VOLTAGE OF SINGLE-PHASE SERIES MOTORS AS A FUNCTION OF THE LOAD.

FIG. 15 represents a single-phase series motor which is provided with a short-circuited compensating winding. Its diagram—neglecting the iron

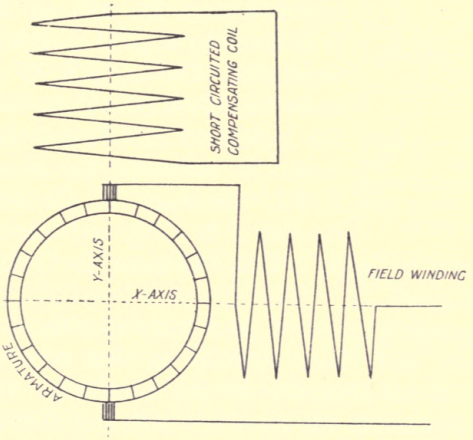

FIG. 15.—Diagrammatic sketch of the compensated series motor.

losses and assuming the armature winding to be completely compensated by the compensating winding—is shown in fig. 16. For a given current OJ,

the E.M.F.'s in the motor are E_1O (ohmic drop), E_2E_1 (produced by the pulsating field in the field winding), and E_3E_2 (produced in the armature winding by the latter's rotation through the field flux). E_3E_2 is proportional to the speed and in phase with the current. E_1O represents an E.M.F. whose direction is from E_1 to O and must be distinguished from the voltage OE_1, which has the same absolute value as E_1O but is opposite in direction. The same remark applies to all other voltages. The three E.M.F.'s E_1O, E_2E_1, and E_3E_2 oppose the flow of current, and so that the current may flow none the less an E.M.F. OE_3 must be applied at the motor terminals. The angle E_3OJ represents the difference in phase between the impressed E.M.F. and the current. At standstill, the impressed E.M.F. necessary to produce the current J is OE_2 and the phase difference is large. OE_2 increases to OE_3 at a certain speed.

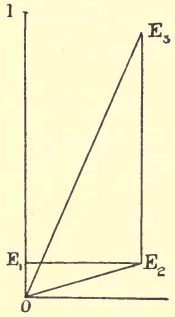

FIG. 16. — Simple diagram of the compensated series motor for a constant current.

In practice, however, the compensation is never complete, and there appears on this account an E.M.F. of self-induction in the armature. OA in fig. 17 represents the armature A.T., and OB stands for the A.T. of the compensating winding. The most convenient way of introducing the armature self-induction is by assuming that only a part ($OA' = v'OA$) of the armature A.T. affects the stator winding. The vector sum of OB and OA′ is OC

and the flux proportional to O C induces in the compensating winding an E.M.F. equal to

(Current in the compens. wind.) × (ohmic resist. of this winding).

Similarly, if $O B' = v'' O B$ represent that part of the A.T. of the compensating winding which affects the armature, then $O E = O A + O B'$ stands for the A.T. which give rise to self-induction in the armature. O E produces a flux which induces in the armature winding a voltage E′O, and the E.M.F. at the motor terminals must now also counterbalance E′O in addition to the other voltages mentioned. O C is at right angles to O B. The manner of calculating the constants v' and v'' will be explained later on.

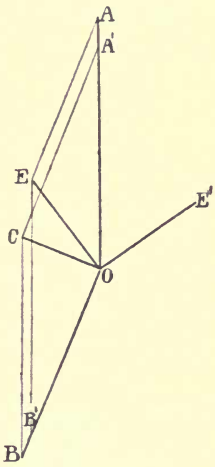

FIG. 17.—Diagram illustrating the compensation of the armature winding of a single-phase series motor.

Fig. 18, a combination of figs. 16 and 17, is the diagram of a single-phase motor with self-induction and negligible iron losses. E_1O and E_2E_1 represent the same quantities as in fig. 16; E_3E_2 is the voltage induced in the armature by the incomplete compensation, while $E E_3$ is the counter-E.M.F. in the armature due to rotation. When the motor is at rest, the impressed voltage is $O E_3$, and at a certain speed U it is O E.

So far, this diagram relates to a certain current only, but it is obvious that as long as the iron remains unsaturated all quantities are proportional to the

current. The angles in fig. 18 remain constant, and all vectors increase in the same ratio as the current. It is seen from this very simple diagram that, at a given frequency of the supply, the power-factor depends only on the speed (increasing with it) and not on the load, except in as far as a change of load affects the speed.

The diagram for a constant terminal voltage is easily developed from that for a constant current

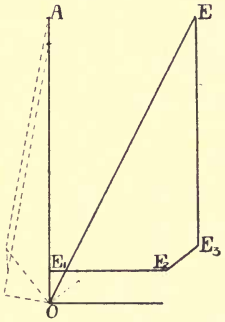

FIG. 18.—Diagram of a single-phase series motor with incomplete compensation and negligible iron losses.

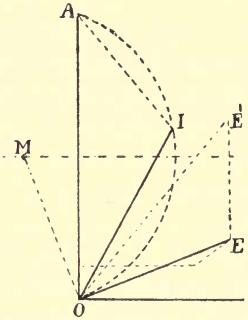

FIG. 19.

when it is remembered that the inverse figure of a straight line is a circle. The voltage O E in fig. 19 ($=$ O E_3 in fig. 18) is sufficient to force a current J ($=$ O A) through the standing armature, but a voltage O E′ ($=$ O E in fig. 18) is required to produce the same current J when the motor is rotating at a speed U. If we keep our terminal voltage constant, however, it means that at the speed U only a current

$$O\,J = \frac{O\,E}{O\,E'} \times O\,A$$

will flow through the armature.

Since the triangles A O J and E′O E are similar, angle A J O = angle E′ E O = constant, *i.e.* the phase-difference between the current O J and the terminal voltage O E is the same as between O A and O E′. The locus of J is a circle, whose centre M is the point of intersection between a perpendicular to O A drawn midway between the points O and A and a perpendicular in O to O E. O E is consequently a tangent to the circle.

Fig. 20 is a repetition of fig. 19, but with the voltage vector upright. O E represents the constant voltage at the motor terminals, O A is the current at standstill (*i.e.* the short-circuit current), and O J stands for the current at a speed U. This speed U is represented by A U, as may be shown as follows: — Referring to fig. 19, U is proportional to E E′, therefore also proportional to E E′/O E, O E being constant as assumed. But this ratio is equal to the ratio J A/O J in figs. 19 and 20. If a line be produced through A parallel to O E, this line being intersected by O J in U, the triangles O A U and O J A are similar and

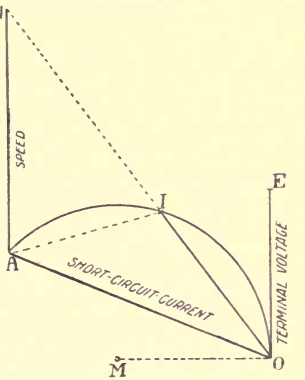

FIG. 20.—Diagram of the single-phase series motor with negligible iron losses for a constant terminal voltage O E.

$$A\,U = O\,A\,\frac{J\,A}{O\,J}.$$

Since O A is a constant, O A $\frac{J\,A}{O\,J}$ (*i.e.* also A U) is a

measure of the speed U. The torque is proportional to the square of O J.

The sparking voltage of a single-phase motor is the vector sum of the reactance voltage and the 'external' voltage induced in the short-circuited coils. E_r, the reactance voltage, is in phase with the armature current as has been already pointed out. If there were no compensating winding, the E.M.F. induced in the short-circuited coils by their rotation through the armature's own flux would also be in phase with the current. But the compensating winding has the effect of retarding this E.M.F., which may now be represented both in phase and magnitude by the vector O A (fig. 21). O A is generally very small in value and may be neglected in ordinary single-phase motors. But the voltage $E_w = O W$, induced in the short-circuited coils by the oscillations of the field flux, is very large. It is evident that this voltage lags 90 degrees behind the field flux, therefore also 90 degrees behind the current. $E_r = O R$ (fig. 21) is calculated from formula (IV.), and O A is proportional to the speed and to the flux in the direction of the brushes. The magnitude and phase of this flux can be determined with the aid of fig. 17, in which O E is the M.M.F.

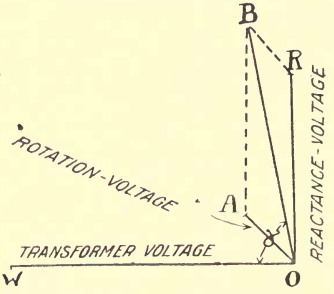

FIG. 21. — Diagram showing the phase relations of the different voltages induced in the short-circuited coils of a single-phase series motor.

which produces the flux in the direction of the brushes.

For calculating $E_w = O\,W$, we can make use of the formula employed in connection with transformers:

$$O\,W = 4{\cdot}44\ N z M\ 10^{-8} \text{ volts,}$$

in which

N = frequency of supply;

z = turns per segment, and

M = flux of one pole.[1]

In a motor with a given flux M, the reactance voltage O R and the 'rotation voltage' O A are proportional to N′ (frequency of rotation), while the 'transformer voltage' O W is proportional to the frequency of supply. All three are proportional to the current in the motor. O B, the vector sum of O R and O A, forms an angle α with O W. In fig. 22, these voltages in the short-circuited coil have been plotted as a function of the current. The curve of the sparking voltage is obtained by adding O B and O W vectorally. Fig. 23 represents the sparking voltage as a function of the load. The forms of the curves in figs. 22 and 23 depend largely on the ratio of O W to O R, and this again depends partly on the flux density in the air-gap. The reactance voltage is smaller, and the 'transformer voltage' is larger, as a rule, the larger this flux density.

In cases in which the transformer voltage is large compared with the reactance voltage, the maximum sparking voltage occurs at starting, because the short-circuit current in the motor then gives rise to

[1] For armatures of the Gramme type M denotes half the flux from one pole.

a strong alternating field which agains induces in the coils a high transformer voltage. The starting torque, however, increases more rapidly with the current than this transformer voltage, and it is clear that by reducing the terminal voltage at starting a sufficient starting torque can be secured without increasing the transformer voltage in the short-circuited coils

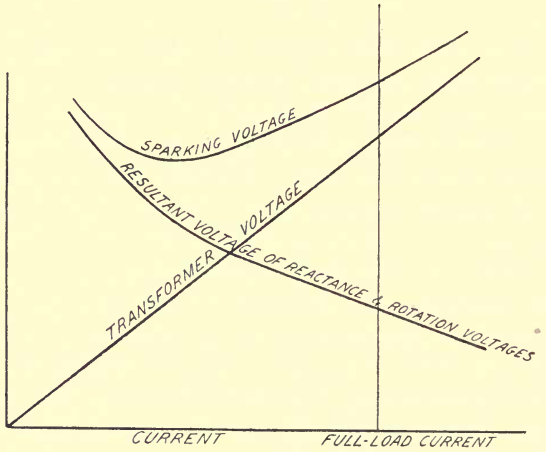

FIG. 22.—The sparking voltage of a single-phase series motor as a function of the current.

unduly. It is, therefore, not the absolute maximum value of the sparking voltage at starting which is of most concern, but rather that sparking voltage which corresponds to a specified torque. If the starting torque is required to be large, then certainly the sparking voltage at starting is of cardinal importance and overshadows all other considerations.

So far, it has been assumed that the various fluxes

were proportional to the M.M.F.'s producing them. As a rule, however, this is not the case, as the iron paths become more and more saturated as the load increases. It is not good practice to keep the flux densities too low, because this favours high reactance voltages. It seems best to design the motor for medium flux densities.

Obviously, fig. 21 is no longer correct when high densities occur. When this takes place, the

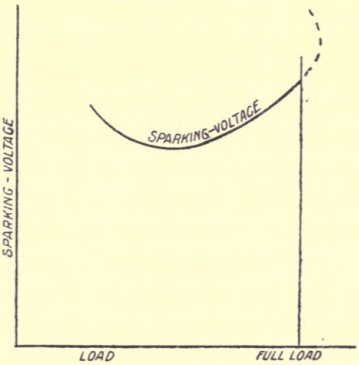

FIG. 23.—The sparking voltage of a single-phase series motor as a function of the load.

characteristic curves of the motor have to be plotted point for point with the aid of the diagram fig. 18. Up to now, we have neglected the M.M.F. of the short-circuited coils. Fig. 24 shows how the diagram of fig. 18 is altered when this M.M.F. is also taken into account. O A represents the armature A.T. and O B stands for the field A.T. Let the A.T. of the short-circuited coils, calculated from equation (V.), be indicated in value and phase by B B′. B B′ is in phase

with the sparking voltage. The resultant field A.T. (and the field flux as long as the magnetic path is not saturated) are then represented by the vector O B′. This resultant flux O B′ gives rise, in the short-circuited coils, to a transformer voltage E_2 E_1 (90 degrees behind O B′) and to a voltage E E_3 due to rotation and in phase with O B′. The dotted lines in

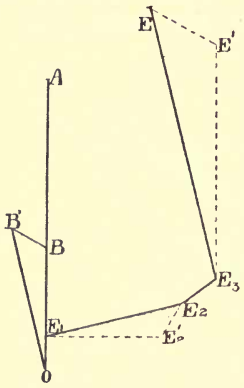

FIG. 24.—Diagram of the single-phase series motor incorporating the reaction of the short-circuited coils upon the field.

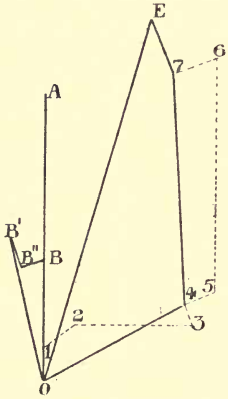

FIG. 25.

fig. 24 correspond to the vectors of fig. 18; they are reproduced to show how the original diagram is altered by including the M.M.F. of the coils under the brush.

It is seen from fig. 24 that, at a given speed, a higher terminal voltage than before is necessary in order to counterbalance the additional effect of this M.M.F. The phase-difference between the impressed voltage O E and the main current has diminished at the same time.

We arrive thus at the following conclusion :—

At a given terminal voltage, the reaction of the short-circuited coils on the field results in a reduction of the speed and in an increase of the power-factor of single-phase series motors.

In fig. 25, the vector B B′ is resolved into the two components B B″ and B″B′. The first is perpendicular to O B′ and corresponds to the transformer voltage E ; the second represents the reactance and rotation voltages. Both vectors correspond, therefore, to the vectors O W and O B in fig. 21.

For a given current, B B″ remains, practically speaking, constant, while B″B′ increases proportionally with the speed. The two components B″B′ and B B″ create fluxes which, in their turn, give rise—partly by induction and partly by rotation—to various voltages in the field and armature windings. All the different voltages are indicated by vectors in fig. 25, which represents the diagram of the single-phase series motor for a constant current but variable speed and terminal voltage. 1–O represents the ohmic drop in the armature[1] and is in phase with the current O A ; 2–1 is the E.M.F. in the armature due to the incomplete compensation of the armature A.T.; the broken line 5–4–3–2 represents the various E.M.F.'s induced in the field winding by different fluxes, and the broken line E–7–6–5 represents the E.M.F.'s induced in the

[1] To be quite correct, the drop of voltage under the brushes ought to be considered separately. Were this done, it would be found that there is a small difference of phase between current and ohmic drop.

armature by its rotation through the various fields concerned.

It will be observed that O–1 and 3–4 are quite independent of the speed, and that 4–5, 5–6, and 6–7 are proportional to the speed, while 7–E is proportional to the square of the speed. Hence, the impressed voltage at the terminals is the vector sum of a constant vector O–4, a vector 4–7 = constant × speed, and a vector 7–E = constant × speed^2.

Such a complicated diagram as that of fig. 25 need not be used for practical purposes; it has been merely developed to show how a change in the conditions affects the action of the motor. The same remark applies to the diagrams to follow.

CHAPTER IX.

THE SPARKING VOLTAGE OF THE REPULSION MOTOR AS A FUNCTION OF THE LOAD.

FIG. 26 is a diagrammatic sketch of the repulsion motor, invented by Professor Elihu Thomson.[1] The stator winding may consist of one coil at an angle to the line connecting the brushes. It is convenient, however, to represent the one actual stator winding by two component windings[2]: one in the direction of the x-axis and producing the field flux M_x, the other in the direction of the y-axis and creating, together

[1] American patent, No. 263,185.

[2] See Lamme, *Transactions of the American Institute of Electrical Engineers*, vol. xxi. p. 87; and Eichberg, *Elektrotechnische Zeitschrift* 1904, No. 4, fig. 33.

with the armature winding, the transformer flux M_y. The brushes are fixed in the y-axis.

At first let us assume that the resistance of the armature circuit is negligible. An E.M.F.—which we will call E_b—is generated in the armature winding

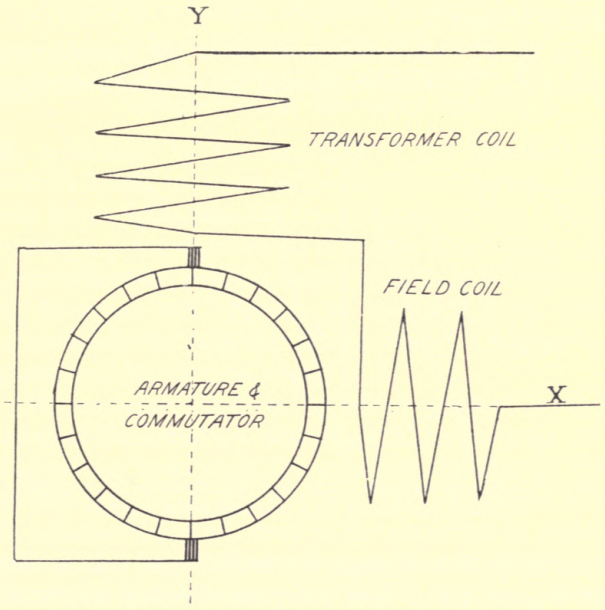

FIG. 26.—Diagrammatic sketch of repulsion motor.

by the armature's rotation through the field flux M_x. If it be assumed—and this assumption is approximately correct—that M_x is in phase with the stator current J, then E_b will also be in phase with the stator current. Since the sum-total of the voltages in the armature must be zero at any moment, it

follows that there must be a second E.M.F. in the armature equal in magnitude but opposite in direction to E_b. This voltage can only be due to the transformer flux M_y as there is nothing else to account for it. M_y, in order to induce the desired voltage in the armature, must lag, therefore, 90 degrees behind the stator current J.

In fig. 27, O A = ampere-turns of the transformer coil and A′ B = ampere-turns of the armature winding.

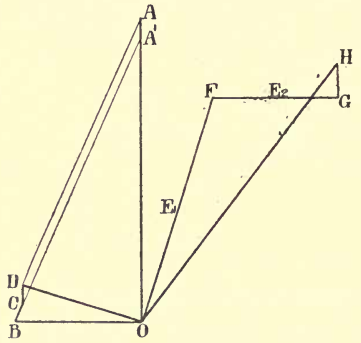

FIG. 27.—Simple diagram of the repulsion motor (with negligible armature resistance) for a constant stator current.

Part of the flux through the transformer coil is lost by dispersion and does not penetrate the armature. Let the transformer-A.T. which act upon the armature be denoted by

$$O A' = v' O A.$$

It is the resultant (O B) of O A′ and A′B which produces, in the direction of the y-axis, an oscillating flux just sufficient to induce in the armature winding an E.M.F. equal and opposite to E_b. The impressed

voltage at the motor terminals is the vector sum of the following three voltages:—

$E_1 = O F$, a voltage necessary to counterbalance the self-induction in the transformer winding;

$E_2 = F G$, a voltage required to compensate the E.M.F. induced in the field coil; and

$E_3 = G H$, a voltage to compensate for the voltage-drop in the stator winding.

E_1 is induced by the resultant of the transformer flux (O A) and that part ($A D = v'' A' B$) of the armature flux which links with the transformer winding. This resultant is represented by O D, and the voltage E_1 must obviously lead the vector of O D by 90 degrees. E_2 is equal and opposite to the E.M.F. of self-induction in the field coil and is thus 90 degrees in advance of the current vector O A.

The complete diagram, in which account is also taken of the armature resistance, is shown in fig. 28. The voltage induced in the armature winding by the alternating flux M_y has now not only to compensate E_b but also the ohmic drop in the armature in phase with the current. Hence the resultant of the transformer–A.T. and the armature–A.T. has to possess not alone a component O B, as before, but also a component B′B in quadrature with the armature current A′B′. B′B is calculated from the equation

$$\frac{B'B}{O B} = \frac{\text{voltage drop in the armature}}{E_b}.$$

The vectors O D, O F, F G, and G H are determined as before. O H is the voltage which has to be

applied to force the current O A through the motor at the given speed. C D is parallel and equal to A′ A. O D is the sum of the four vectors O B, B B′, B′C, and C D. Similarly, we may resolve O F into the four components O M, M N, N Q, and Q F. The broken line O B B′C D is similar to O M N Q F, but turned round 90 degrees. The meaning of each vector is clear from its direction. Q F, F G, and G H—at a given stator current—are entirely in-

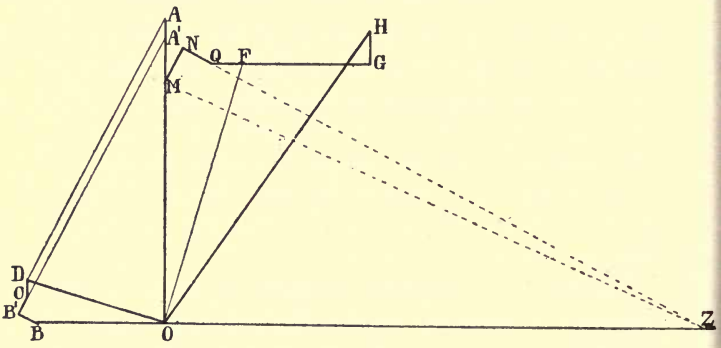

Fig. 28.—Diagram of the repulsion motor for a constant stator current, account being taken of the armature resistance.

dependent of the speed; O M is exactly proportional to the speed, while M N and N Q increase with the speed but not in exact proportion. Draw B O and N Q to meet in Z. Then

Triangle Z M N is similar to triangle A′B B′;

and, since the *form* of these triangles is independent of the motor speed, the angles of triangle Z M N always remain the same. The final diagram of the repulsion motor for a con-

stant current may now be drawn as shown in fig. 29, in which O A = stator current, O B = self-induction in the stator (= Q F + F G in fig. 28), and B C = ohmic drop of voltage in the stator winding. Draw a perpendicular to O A through C and make C Z = O Z in fig. 28, and construct the triangle Z M N Q by making M N = ohmic drop of voltage and N Q = E.M.F. of self-induction in the armature. At standstill the triangle is in the position Z C N′Q′, and O Q′

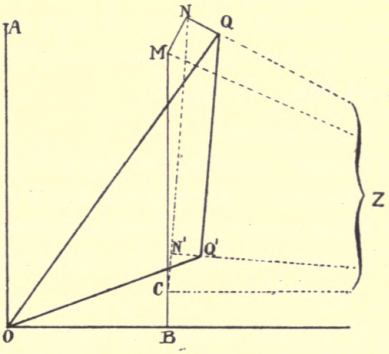

FIG. 29.

represents the requisite voltage to produce the current O A in the stator when the motor is at rest. As the motor speeds up, the triangle changes its size and position, Z remaining at its place while the point M shifts along a line parallel to O A. Z M obviously increases steadily with the speed, and, since the triangles in every position are all similar to one another, the other sides of the triangle, viz. M N and N Z, increase in the same ratio. Point Q interests us most because O Q represents the terminal

voltage, and we can shorten the discussion on the diagram by considering the triangle Z M N.

It is not difficult to see that the locus of Q is a straight line forming the same angle with M C as Q Z does with M Z. Q′Q, like C M, is a measure of the speed, for

$$Q'Q = \frac{QZ}{MZ} CM = \text{constant} \times CM.$$

The simplest form of the diagram of the repulsion motor for a given stator current is shown in fig. 30. It takes no account, however, of iron and friction losses, and of the reaction of the short-circuited coils upon the field flux. In fig. 30:

O A = Stator current;
O Q′ = Voltage required to force O A through the stator when the motor is at rest;
Q Q′ = A distance proportional to the speed U; and
O Q = Voltage at the stator terminals at the speed U.

It is now a simple matter to develop from the constant-current diagram (fig. 30) the diagram which obtains when the voltage at the stator terminals is kept constant (fig. 31). Let O E represent this constant terminal voltage and let O A denote the short-circuit current at this voltage. The difference in phase between the two must be represented in both figures by the same angle—*i.e.* angle A O Q′ in fig. 30 = angle A O E in fig. 31.

If it is desired to find the stator current for any speed—say for the speed corresponding to the voltage O Q in fig. 30—draw the triangle O A J (fig. 31) similar to the triangle O Q Q′ (fig. 30). Then

O E and O J form the same angle as O Q and O A, and, furthermore, $\frac{O E}{O J}$ (fig. 31) $= \frac{O Q}{O A}$ (fig. 30).

Considering that the angle A J O (= angle Q Q′O) is constant, it follows that the locus of the point J is a circle whose centre M is below the line O X (O X is at right angles to O E). The angle M O X = angle between O A and Q′Q in fig. 30. The motor

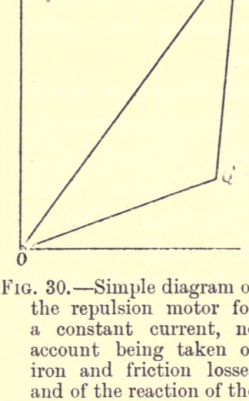

FIG. 30.—Simple diagram of the repulsion motor for a constant current, no account being taken of iron and friction losses and of the reaction of the commutating coils.

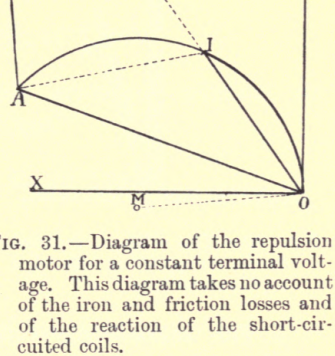

FIG. 31.—Diagram of the repulsion motor for a constant terminal voltage. This diagram takes no account of the iron and friction losses and of the reaction of the short-circuited coils.

speed is proportional to A B (B being the point of intersection between O J and the perpendicular to O M through A), as can be proved as follows:—

We know that the speed is proportional to $\frac{Q′Q}{OQ′}$ (fig. 30), hence also proportional to $\frac{AJ}{OJ}$ (fig. 31) and proportional to $\frac{AB}{OA} = \text{constant} \times AB$.

It would not be difficult to develop other diagrams from fig. 31 to show the effect of varying the conditions in all kind of manners; but such complicated diagrams are of little practical value. Even the diagram fig. 31 is not simple enough, and when it is remembered that fig. 31 is based on the assumption that the fluxes are proportional to and in phase with the M.M.F.'s, it becomes evident that it is preferable in many cases to make use of the very simple diagram of fig. 27 and to calculate the current for a few points.

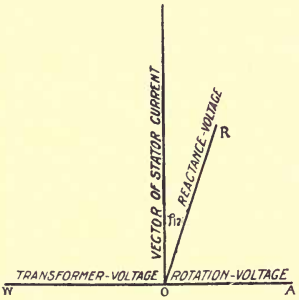

FIG. 32.—The voltages induced in the commutating coils of a repulsion motor.

These diagrams enable us to investigate how the sparking voltage of a repulsion motor varies with the load. The different voltages in the coils short-circuited by the brushes are indicated in fig. 32. They are:—

1. The transformer voltage O W, which is proportional to and 90 degrees behind the flux M_x. O W depends on the frequency of supply but not at all on the motor speed.

2. The reactance voltage O R, proportional to and in phase with the armature current and proportional to the speed.

3. The voltage O A due to the rotation of the short-circuited coils through the field M_y. O A is proportional to M_y and to the armature speed.

On the assumption that both the field flux M_x

and the transformer flux M_y follow the sine law,[1] we find:

$$O\,W = 4{\cdot}44\, z\, N\, M_x\, 10^{-8} \text{ volts, and}$$
$$O\,A = 4{\cdot}44\, z\, N' M_y\, 10^{-8} \text{ volts.}$$

Now, M_x and M_y are, as we already know, not independent of one another. They are connected by the fact that their resultant has to be of such magnitude and phase as to induce in the armature winding an E.M.F. to compensate the ohmic drop in the armature. This drop, however, is usually small.

M_x gives rise, in the armature winding, to an effective 'rotation voltage'

$$= \frac{4}{\sqrt{2}}\, s\, N' M_x\, 10^{-8} \text{ volts,}$$

and M_y produces in the armature winding an effective 'transformer voltage'

$$= \frac{4{\cdot}44}{\pi/2}\, s\, N\, M_y\, 10^{-8} \text{ volts.}$$

Hence, very nearly,

$$N' M_x = N\, M_y, \quad \text{or} \quad M_y = \frac{N'}{N} M_x.$$

This value introduced into the equation for O A gives

$$O\,A = 4{\cdot}44\, z\, \frac{N'^2}{N} M_x\, 10^{-8} = \left(\frac{N'}{N}\right)^2 O\,W.$$

O A and O W have opposite signs, and their vector sum is equal to

$$O\,A + O\,W = O\,W\left[1 - \left(\frac{N'}{N}\right)^2\right].$$

At synchronous speed

$$N' = N, \quad i.e. \quad O\,A + O\,W = 0,$$

[1] It is shown in Appendix I. that this is not actually the case.

and the sparking voltage is simply equal to the reactance voltage.

At speeds below synchronous speed, O W is larger than O A, and at speeds above synchronous speed O A predominates. The sparking voltage and its phase-angle with the stator current are different, therefore, at every speed.

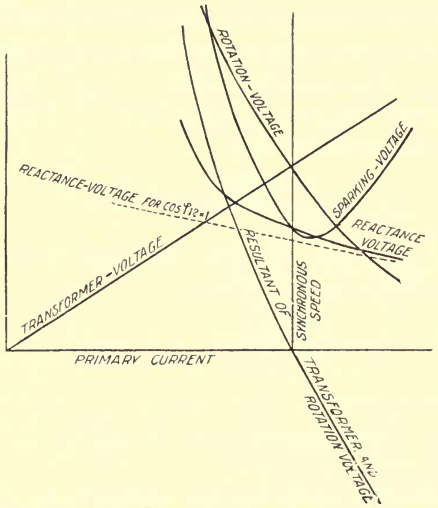

FIG. 33.—Diagram indicating the variation of the sparking voltage of a repulsion motor with the stator current.

It is also interesting (see figs. 33 and 34) to plot the sparking voltage of the repulsion motor against the stator current and the load respectively. To this end the diagram of fig. 27 may be made use of. We assume various values of current and obtain then from fig. 27 the corresponding speeds. If the data of the motor are given, O W and O A are easily calculated from the equations stated above.

The only quantity which now remains to be determined is the reactance voltage:

Let the angle between stator and armature current (=angle B A′O in fig. 27) be denoted by ϕ_{12}. The armature current is

$$J_2 = \frac{J_1}{\cos \phi_{12}}, \text{ and } \quad \tan \phi_{12} = \frac{O\,B}{O\,A'} \text{ (fig. 27).}$$

It will be shown later on that at synchronous speed

$$\frac{O\,B}{O\,A'} = \frac{\text{turns of field coil}}{\text{turns of transformer coil}}.$$

If this ratio be denoted by tan α, we have

at synchronous speed: $\tan \phi_{12} = \tan \alpha$;

at all other speeds: $\tan \phi_{12} = \frac{N'}{N} \tan \alpha$.

Hence, $$J_2 = J_1 \sqrt{1 + \left(\frac{N'}{N} \tan \alpha\right)^2}.$$

The reactance voltage is in phase with J_2, and both its magnitude and phase are fixed when the angle ϕ_{12} is given. Figs. 33 and 34 have been determined in this way. In fig. 33 the absolute values of the components of the sparking voltage have been plotted, but their phase relations have not been shown.

Of course, the above equations do not hold good when the iron paths become saturated. When this occurs, the characteristic curves have to be plotted point for point.

The reaction of the short-circuited coils upon the field plays a more prominent part in repulsion motors than in single-phase series motors because its effect with regard to the power-factor is good or bad according to the speed. For speeds below

synchronous speed, the sparking voltage lags behind the armature current, and, as a rule, also behind the stator current (fig. 32), and this has the effect of increasing the power-factor. But when the speed exceeds synchronous speed, the sparking voltage leads the stator current and thus has the effect of shifting the field flux forwards.

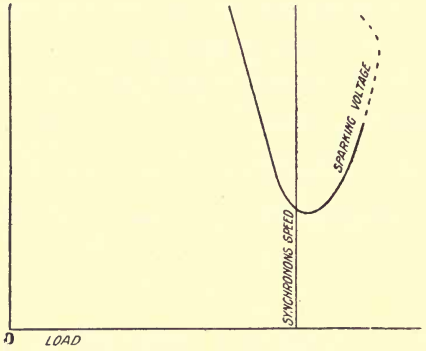

FIG. 34.—Diagram showing how the sparking voltage of a repulsion motor varies with the load.

Fig. 35 is the diagram of the repulsion motor at speeds below synchronism, account being taken of the reaction of the short-circuited coils upon the field; the dotted lines correspond to the vectors of fig. 28.

O A = Ampere-turns of the transformer coil, and
O S = Ampere-turns of the field coil.

The reaction of the short-circuited coils, calculated as explained above, is in phase with the sparking voltage, the magnitude and phase of which are computed as set forth in the preceding pages. S S′ in fig. 35 represents the reaction in question, and the

resultant field flux of the motor is given, both in value and phase, by the vector O S′. O B is at right

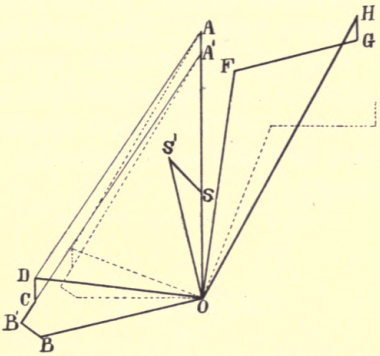

FIG. 35.—Diagram of a repulsion motor below synchronous speed, showing the effect due to the reaction of the coils undergoing commutation.

angles to and also proportional to O S′ if the arma-

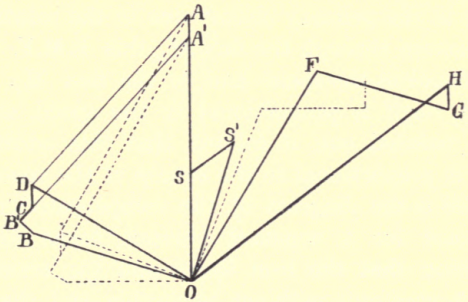

FIG. 36.—Diagram of a repulsion motor above synchronous speed, showing the effect due to the reaction of the coils undergoing commutation.

ture resistance be neglected. The other points, viz. B′, C, D, and F, are determined as before (fig. 28).

F G is perpendicular to O S′, while G H remains as before.[1]

Fig. 36 is the same diagram, but for speeds higher than synchronous speed. The sparking voltage, determined with the aid of these more complete diagrams, differs to a certain extent from the values obtained by means of the simple diagram, and, if great accuracy is aimed at, a correction has to be made accordingly.

CHAPTER X.

THE COMPENSATED REPULSION MOTOR.[2]

INSTEAD on the stator, the field winding is on the armature, being one with the ordinary armature winding (fig. 37). An additional E.M.F. now appears in the primary circuit due to the rotation of the armature through the flux M_y, this E.M.F. being proportional to the speed and to the flux M_y. But since M_y itself is proportional to the speed, as previously pointed out, it follows that this additional E.M.F. is proportional to the square of the speed.

Fig. 38 is the diagram of the compensated repulsion motor for a constant current, based on the assumption that the iron losses and the resistance of the short-circuit path are negligible and that the fluxes are proportional to the respective ampere-

[1] To be quite correct, the voltage-drop under the brushes ought again to be considered separately.

[2] Invented simultaneously by M. Latour in France, and Herren Winter and Eichberg in Germany.

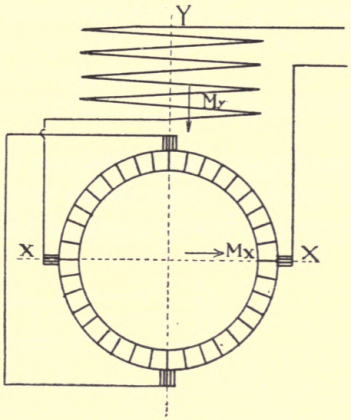

FIG. 37.—Diagrammatic sketch of a compensated repulsion motor.

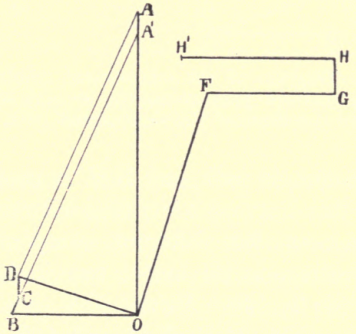

FIG. 38.—Simple diagram of the compensated repulsion motor for a constant primary current.

turns. It is identical with the diagram of the repulsion motor (fig. 27) with the exception of the vector H H′.

We have $$H H' = \left(\frac{N'}{N}\right)^2 G F.$$

As soon as the motor speed increases a little beyond synchronous speed the power-factor becomes unity.

In fig. 39 account is also taken of the resistance

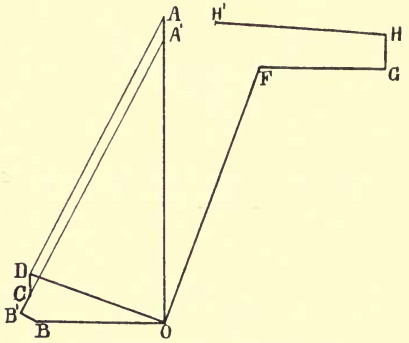

FIG. 39.—Diagram of the compensated repulsion motor for a constant primary current, account being taken of the resistance of the short-circuit path.

of the short-circuit path, and H H′ is parallel to O B′. Otherwise, this diagram is identical with that of fig. 28.

For clearness, the different components of the impressed voltage at the motor terminals have been re-plotted in fig. 40. Vector O F has four components, viz. O M, M N, N Q and O F, and vector H H′ is the resultant of H J and J H′.

O M is the component in phase with the primary current, being proportional to the speed.

M N is the component required to counterbalance the ohmic drop of voltage in the short-circuit.

N Q is required to compensate the self-induction of the short-circuited winding.

Q F counterbalances the self-induction of the stator winding.

F G has to compensate the voltage induced in the field winding by the field flux.

It must be remembered that the field winding is

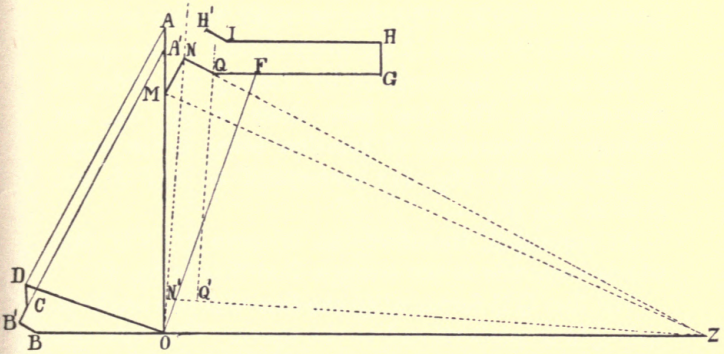

FIG. 40.—Diagram of the compensated repulsion motor for a constant primary current.

on the armature and that the field flux is in the direction of the x–axis.

G H serves for forcing the primary current against the total resistance of the primary circuit, the total resistance being equal to resistance of stator winding + resistance of armature winding + contact resistance of the brushes.

H J and J H′ have to counterbalance the E.M.F.'s produced by rotation through the field M_y. H J and

J H′ are parallel to and proportional to O B and B B′ respectively.

Z is again the point of intersection between the perpendicular to O A through O and the line N Q. The triangle Z M N retains its shape for all speeds, but it varies in size so that O M is a measure of the speed. The locus of N is a straight line through O.

Angle M O N = angle M Z N = angle B′O B = angle H′H J = γ. The locus of Q is also a straight line.

When the motor is at rest, the triangle is in the position O N′Q′Z, N′Z being at right angles to O N, and both N′N and Q Q′ are proportional to the speed.

Fig. 40 may be simplified by altering the order of the vectors. This has been done in fig. 41, the voltages independent of rotation being taken first.

O B and B C (fig. 41) correspond to Q F G and G H in fig. 40.

At standstill, therefore, the triangle Z C N′Q′ (fig. 41) corresponds to the triangle Z M N Q in fig. 40.

The line Q′Q forms with the line O A the angle γ, and Q′Q is proportional to the speed. The triangle E F Q corresponds to the triangle H′J H (fig. 40). Q F is perpendicular to O A and proportional to the square of the current, therefore also proportional to QQ'^2. Angle F Q E = γ and E F is parallel to Q Z. Fig. 42 is a simplification of fig. 41, being obtained by omitting all unessential vectors. The locus of point E is a parabola as can be shown as follows:— Assuming the abscissæ to be measured along Q′Q, and the ordinates along a perpendicular to Q′Q, then E Q is directly an ordinate. Draw F F′ parallel to Q′Q, then

$$EQ = EF' + F'Q.$$

Since $F'Q = F\,Q \cos \gamma$, and $F\,Q$ is proportional to the square of the speed, it is evident that $F'Q$ too is proportional to the square of the speed. Now, triangle $F\,F'E$ is similar to triangle $Q\,Q'Z$ (fig. 41) because the sides of the one triangle are parallel to the sides of the other.

Therefore $$\frac{E\,F'}{F\,F'} = \frac{Q'Z}{Q\,Q'}, \quad \text{or} \quad E\,F' = Q'Z \times \frac{F\,F'}{Q\,Q'}.$$

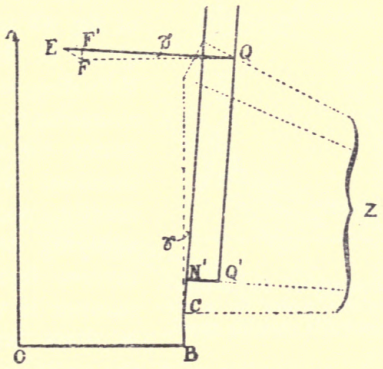

FIG. 41.—Same as fig. 40, but with the vectors in a different order.

But $F\,F'$ is proportional to $Q\,Q'^2$ and $Q'Z$ is constant; hence $E\,F' = \text{constant} \times Q\,Q'$ The general equation of E is, therefore,

$$y = C_1 x^2 + C_2 x\,; \text{ in which } E\,Q = y \text{ and } Q\,Q' = x.$$

This is the equation of a parabola whose origin is displaced from Q′ in the directions of both the x-axis and the y-axis.

We are now enabled to construct the locus of E (for a constant current and variable speed) in the

following simple manner:—Mark a point G on Q E so that Q G = F′E, then Q′G = constant × speed, and G E = constant × speed². The two constants depend on the data of the motor and can be calculated when these are known. The parabola of fig. 42 has been determined in this manner.

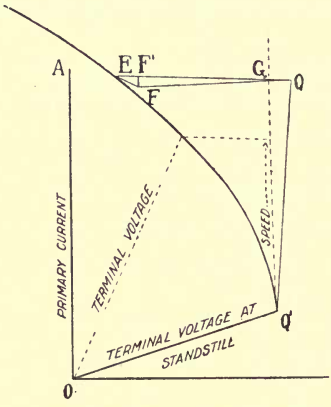

FIG. 42.—Diagram of the compensated repulsion motor for a constant current and variable speed.

Since all the vectors of fig. 42 are proportional to the current, the diagram of the compensated repulsion motor for a constant terminal voltage is readily obtained by applying the law of inverse figures. The manner in which this is done was shown previously in connection with the ordinary repulsion motor (fig. 31).

Fig. 43 is the diagram obtained in this way; but it is preferable to use fig. 42, as the parabola is lost by the application of the method of inverse figures.

SPARKING VOLTAGE OF THE COMPENSATED REPULSION MOTOR

It is notable that the commutation of the coils short-circuited by the brushes in the x-axis (which brushes convey the primary current to the armature) depends only on the reactance voltage, no other

E.M.F. appearing in these coils.[1] The voltages induced in the coils under the brushes in the y-axis are the same as in the case of the ordinary repulsion motor, so that fig. 32 applies, as it stands, to the compensated repulsion motor also. As soon, therefore, as the current and speed have been determined from figs. 42 or 43, the sparking voltage under the short-circuited brushes can be computed in the

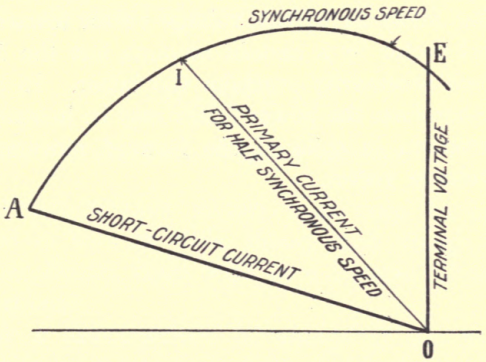

FIG. 43.—Diagram of the compensated repulsion motor for a constant terminal voltage.

manner explained in connection with the repulsion motor. The same formulae also apply for determining the different components. The reaction of the short-circuited armature coils on the field has the same effect as in a repulsion motor, *i.e.* the voltage vector is shifted forwards for speeds higher than synchronous speed, and is shifted backwards

[1] It is even possible to compensate the reactance voltage by suitable means, so that the commutation under the brushes in the x-axis may be rendered perfect.

towards the current vector when the motor speed drops below synchronous speed. In the latter case the power-factor always increases, while in the former case the power-factor may grow better or worse according to the conditions. Take as an example the case in which the voltage vector is found by diagram to lag 20 degrees behind the current vector. If, now, the reaction of the commutating coils upon the field be taken into account, the result will be a shifting forwards of the voltage vector towards the current vector by a certain amount, but the former need not necessarily overtake the latter. In such a case, therefore, the power-factor may be increased by the reaction in spite of the forward movement of the voltage vector.

PART II.—THE DESIGN OF COMMUTATOR MOTORS.

CHAPTER XI.

CONTINUOUS-CURRENT MOTORS.

CONTINUOUS-CURRENT motors need only be briefly referred to here as their theory and design have been excellently dealt with in a number of recent publications.[1]

It is a characteristic advantage of d.c. motors that they may be successfully designed for higher flux densities than any other type of commutator motor. For this reason they can be built smaller and cheaper than single-phase commutator motors of the same output and speed, and their sparking voltage too is smaller. There is an economic limit, of course, to the flux density that may be employed, as, after a certain point has been reached, the

[1] *Electric Generators*, by Messrs Parshall & Hobart. *Electric Motors*, by H. M. Hobart (London : Whittaker & Co.), 12s. 6d. *The Dynamo*, by C. C. Hawkins & F. Wallis (Whittaker & Co.), 3rd edition, 15s. *Dynamo-Electric Machinery*, by S. P. Thomson (E. & F. N. Spon), 7th edition, 30s. *Die Gleichstrommaschine*, by E. Arnold, vol. i., 16 marks ; vol. ii., 18 marks.

requisite field copper increases out of all proportion to the gain in flux density. The density at the face of the pole-shoe varies from 5,000 lines per sq. cm. in small motors to 9,000 lines per sq. cm. in larger motors, and in traction motors densities up to 12,000 lines per sq. cm. are occasionally adopted.

The advantages to be gained by increasing the flux density in d.c. motors are illustrated by the following example:—Consider a 10-H.P. motor (lap-wound) having a sparking voltage of three volts. Assume that it is possible to increase the flux density by 10 per cent. by reducing the air-gap or altering the dimensions of the teeth, but without changing the motor otherwise. The speed and sparking voltage of this modified motor are 10 per cent. less than before. If the output of the machine was previously limited by considerations of sparking, it is seen that the motor may now be loaded to the extent of 11 H.P., the commutation being as good as in the 10-H.P. motor running at a 10 per cent. higher speed. The gain is thus 20 per cent. Of course, the copper and iron losses have also increased, the former by 20 per cent. and the latter by about 6 per cent.; but in open and semi-enclosed motors for voltages between 200 and 600 volts it is always the commutation and not the temperature rise which limits the output. This point is of some importance, as various authors have recently averred the contrary.

It is to be regretted that the possibility of increasing the motor output by providing a proper ventilation to keep down the temperature is not

yet sufficiently appreciated.[1] It is not too much to say that the output of most of the open and semi-enclosed motors on the market could be doubled by providing them with efficient ventilating ducts (both vertical and horizontal) in the armature and by suitably designing the commutator and the field coils.[2] This remark refers, of course, to motors whose outputs are limited by the rise in temperature but not by considerations of sparking.

In fully-enclosed motors and in certain low-voltage machines, on the other hand, the output certainly is limited by considerations of temperature. In such cases the question of high flux densities loses in importance; indeed, the densities adopted in fully-enclosed motors are often from 5 to 10 per cent. less than those employed in semi-enclosed motors of the same output.

In fixing the air-gap density, dynamo designers are often guided by the old rule according to which the field A.T. ought not to be less than from 3 to 5 times the armature A.T. per pole. The opinions regarding this ratio of field A.T. to armature A.T. have materially changed of late, especially since the appearance of Parshall & Hobart's *Electric Generators*. Indeed, it is possible to design good and cheap machines in which the armature–A.T. per pole are equal to or even higher than the field–A.T.

Large low-speed motors and also low-speed com-

[1] Mr H. M. Hobart especially has drawn attention to this point.

[2] The design adopted by the Phœnix Dynamo Manufacturing Co. (see *The Electrician*, Oct. 20, 1905, p. 31) is a good example of efficiently ventilated field coils.

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pound generators ought to be designed for a large number of armature–A.T., while high-speed motors and generators are preferably designed with a smaller number of armature–A.T. per pole.

Since the output of a dynamo is equal to

constant × speed × armat. A. T. × flux entering the armat.,

it is clear that the larger the number of armature A.T. the smaller the armature for a given output, assuming of course that the air-gap density is the same in each case. This last assumption is not quite correct, because the ratio of tooth width to slot width is less in the smaller machine than in the bigger machine. Nevertheless, the gain is substantial.

It is not pretended that a large number of armature–A.T. per cm. periphery (called 'armature-strength') is better from the commutation point of view—the reverse is the case, in fact—but it is maintained that in certain types of dynamo, alluded to above, it is cheaper to keep down the sparking voltage by other means than reducing the 'armature-strength.' These considerations relate only to motors and generators which have but one turn per segment. They would no longer be correct if—to adapt the dynamo for a greater 'armature-strength'—it were necessary to increase the turns to two.

The assertion that the choice of the armature strength depends on the motor speed cannot be maintained if the armature has more than one turn per segment. In such cases the influence of the speed disappears entirely. The capacity of a motor, originally designed with two turns per segment, can

be doubled, for instance, by halving the turns per segment and doubling the speed. The commutation will be approximately equally good in the two cases. If, however, the motor has only one turn per segment to start with, the commutation will grow worse as the speed increases, and there is a certain speed for every load at which excessive sparking cannot be prevented by ordinary means.

The kind of armature winding used has also a bearing upon the armature-strength it is well to employ. Motors of less than 60 H.P. are usually of the 4-pole type, the armatures being provided with a wave winding or a lap winding. Each winding has its advantages and drawbacks.

One of the chief advantages of wave windings is that the number of armature conductors and the cost of winding an armature is reduced and that not so much room is taken up by the insulation as in the case of a lap winding. The latter point is of considerable importance in small 500-volt motors. The number of turns per segment, and, consequently, also the reactance voltage, are only half what they would be in a lap-wound armature. But it may be shown theoretically, and has been verified experimentally, that, despite the great reduction in reactance voltage, the sparking is only little better (from 10 to 30 per cent.) than in the corresponding lap-wound armature. This is due, as already indicated in a previous chapter, to the fact that the sparking voltage per brush has only decreased slightly.

It thus follows that the permissible reactance voltage is much higher in a lap-wound armature than in a wave-wound armature. Nothing can be

gained by adopting a wave winding where a lap winding with one turn per segment may be used instead. In such cases, wave windings are to be avoided as much as possible.

It is well known that an asymmetric distribution of the field flux around the armature does not lead to an unequal distribution of current in a wave-wound armature as would happen were the armature lap-wound. This feature was considered to constitute a great advantage of wave windings, and lap windings were badly neglected for a time. But it had been forgotten that inequalities in the contact resistance of the brushes or asymmetrical brush positions could also result in unequal currents flowing through the different circuits of the armature. True, the latter influences also operate in a lap winding, but then we have the following two simple and cheap remedies:—

(1) A large number of armature A.T. per pole.

(2) Equalising connections.

The first and second means are of no avail for wave-wound armatures; the second means may be employed for a composite armature winding, but is not as simple and positive as in lap windings.

In small motors a large number of armature A.T. is not always possible, and the small air-gap of such motors fosters an asymmetrical distribution of the field flux around the armature, and the brush resistance forms a smaller part of the armature resistance than in large machines. For all these reasons it is advantageous to employ the wave winding in small motors, while the lap winding is best in all respects for larger motors.

The following remedies have been tried with some success for improving the commutation of high-speed generators:—(1) Decreasing the inductance of the short-circuited coils by using a smooth armature; (2) Déri-winding or auxiliary poles.

By using the first means the reactance voltage may be reduced to one-half, or, in some cases, to one-third. It is essential to keep the armature A.T. low, and the peripheral width of the pole-shoe must be small compared with the pole-pitch, otherwise the voltage induced in the short-circuited coils by the end connections will form a large part of the sparking voltage. When employing this means it is also well to adopt a long armature (rather than a short armature with a large diameter), as the inductance of the end-connections is now of much more importance than in normal dynamos.

But the second means of reducing sparking is much more effective, and is accomplished by the Déri-winding and by auxiliary poles in much the same way. In both cases, in fact, the armature flux is over-compensated, and a voltage is induced in the short-circuited coils which promotes good commutation. While auxiliary poles, however, only help to suppress sparking, the Déri-winding also neutralises the effect of the cross–A.T. of the armature.[1] Since the 'commutating-flux' (*i.e.* the 'external' flux which assists commutation) ought, as far as possible, to increase proportionally with the motor current, the saturation of the teeth and all other iron parts in the path of this flux must be kept low. In cases

[1] This neutralisation exists no longer when the reactance voltage is high. (See Seidener, *Zeitschrift für Elektrotechnik*, No. 22, p. 355.)

where it is impossible to comply with this condition, the number of the A.T. of the compensating winding is sometimes regulated, when the load changes, by automatic means or by hand.

It can be proved[1] that the reactance voltage is approximately proportional to

$$\frac{\text{average voltage per seg.} \times \text{armat. A.T. per cm. periphery.}}{\text{mean air-gap density}}$$

In order to ensure that the teeth are not saturated by the 'commutating-flux,' care must be taken that the reactance voltage is considerably smaller than the avarage voltage per segment. At a given reactance voltage, this average voltage per segment depends obviously on the ratio

$$\frac{\text{armature A.T. per cm. periphery}}{\text{mean flux density in the air-gap}}.$$

When employing the Déri-winding or auxiliary poles, care must, therefore, be taken that the flux density in the gap under the main poles is large and that the 'armature-strength' is comparatively small. In practice, however, the air-gap density is frequently kept low in order to reduce the iron losses, and a large number of armature A.T. is chosen so as to get a small armature. Thus just the reverse is done of what is demanded by the above rule, and so it comes about that a high saturation in the path of the 'commutating-flux' cannot always be prevented.

Motors with compensating devices are more ex-

[1] This relation is due to McLeod. (See *Technics*, 1903, p. 445.)

pensive than ordinary motors, and it must be decided from case to case whether the application of compensating devices is justified or not.

The calculation of the necessary ampere-turns of the compensating winding is very simple:—

Before any flux can be produced in aid of the commutation, the armature's own flux must be neutralised, *i.e.* the compensating winding must, first

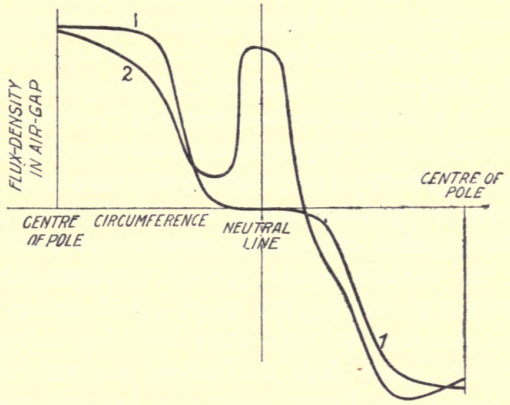

FIG. 44.—Distribution of flux around the armature of a d.c. motor fitted with auxiliary poles. Curve 1 corresponds to no-load and Curve 2 to full-load.

of all, have as many A.T. as the armature per pole. Then the compensating winding must have an additional number of A.T. sufficient to produce a flux through the armature which will induce in the short-circuited coils an E.M.F. equal and opposite to the reactance voltage. These additional A.T. may thus easily be computed from the reactance voltage to be compensated and from the dimensions of the motor.

If the number of armature A.T. per pole be large, it can happen that the auxiliary poles must be provided with more A.T. than the main poles, and the consequence is a considerable increase in the active materials and in the field losses. Not only that, but the iron losses in the teeth have also grown, owing to the altered distribution (see fig. 44) of the magnetic flux around the armature. On the other hand, compensating windings conduce to a reduction of the commutator losses, as the commutation has become well-nigh perfect.

The Déri-winding has this advantage over the auxiliary poles that the number of A.T. on the main field coils can be kept comparatively small since, as has already been pointed out, the armature-cross turns are completely compensated.

The question of sparking plays a very prominent part in variable-speed motors employing shunt regulation for varying the speed. Such motors have to be designed so that their sparking voltage at maximum speed does not exceed a certain value, and that at the lowest speed the temperature rise keeps within the specified limit.

CHAPTER XII.

THE SINGLE-PHASE SERIES MOTOR.

In single-phase series motors the chief considerations are power-factor, commutation, and temperature rise. In order to obtain a high power-factor, the voltage induced in the field winding ought to be

small compared with the voltage across the armature. This is achieved (1) by employing a small number of field turns compared with the armature turns in series between the brushes; and (2) by adopting a high frequency of rotation compared with the frequency of supply. It thus follows that a comparatively low supply frequency ought to be chosen when the output and speed of the motor are fixed, and that—when the supply frequency is given—the motor speed ought to be high in comparison. To reduce the field turns as far as possible, the air-gap and the saturation of the respective iron paths must be kept down. If, for instance, the turns of the field winding of a certain motor are a-times smaller than those of the armature winding in series between the brushes, and if the frequency of rotation is b-times that of the supply current, then the power-factor is, approximately,

$$\cos\phi = \frac{1}{\sqrt{1+\left(\frac{1}{ab}\right)^2}}.$$

This equation is obtained by reference to fig. 16, the ohmic drop O E_1 being neglected.

For $a=1$	and	$b=1$		$\cos\phi=0{\cdot}707$
„ $=1{\cdot}5$	„	$=1{\cdot}5$		„ $=0{\cdot}915$
„ $=2$	„	$=2$		„ $=0{\cdot}970$

The equation for $\cos\phi$ just quoted takes account neither of the self-induction of the armature nor of the field leakage. The latter is insignificant, however, on account of the small M.M.F. involved. While the leakage-coefficient of continuous-current machines ranges from 1·10 to 1·20, it varies for

normal single-phase series motors from 1·03 to 1·08. It implies that from 3 to 8 per cent. more lines of force are linked with the field winding than with the armature winding. Its effect upon the power-factor can be taken account of by substituting in the above equation a/f for a, f denoting the leakage-coefficient.

Of greater importance is the self-induction of the armature, all the more so as the number of armature A.T. is kept large for other reasons. The dispersion of the armature may be approximately arrived at as follows:—We assume that—in the case represented by fig. 17—the compensating winding has no resistance, and obtain on this basis the diagram fig. 45, in which the points O, C, D, E, A′, A, B′, and B lie on a straight line. O and C coincide in this diagram.

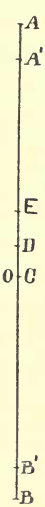

FIG. 45.

O E represents the A.T. which conduce to the dispersion, assuming the reluctance of the dispersion path to be the same as the reluctance of the path of the main armature flux. Let the ratio O E/O A be denoted by σ_k; σ_k corresponds to the dispersion-coefficient σ, well known in connection with the theory of three-phase motors. This coefficient σ_k can be calculated from the formula

$$\sigma_k = \frac{C_1}{H^2} + C_2 \frac{\delta}{l},$$

in which C_1 and C_2 are constants depending only on the type of winding employed, especially on that of

the compensating device. This formula is borrowed from a paper on the "Dispersion-coefficient of Three-phase Motors," read by Herrn Behn-Eschenburg before the Institution of Electrical Engineers.[1] It is quoted here in a slightly modified form to conform to single-phase working. Behn-Eschenburg's original formula for this coefficient contains a third term, which takes account of the dispersion due to the slots; but this third term, being of small importance, has been left out in our case. According to Dr Behn-Eschenburg, $C_1 = 3$ and $C_2 = 6$. It has been pointed out by others (Hobart, Guilbert), however, that the value assigned to C_1 is decidedly too small, and we will assume the correct values to be $C_1 = 4{\cdot}5$ and $C_2 = 6$.[2]

H = (average number of armature slots per pole plus average number of slots of the short-circuited compensating winding per pole) 2.

δ is the air-gap in cm., and

l is the axial length of the armature core in cm.

In the case where the air-gap is the same all around the armature, the E.M.F. induced in the armature winding by $OE = \sigma_k \times OA$ can be taken into account by assuming that the field turns are increased by a number of turns equal to

$\sigma_k \times$ (armature-turns in series between the brushes) [2]

or, what amounts to the same thing, we can substitute

[1] See *The Electrician*, vol. lii. pp. 525, 647, 741, 821, and 954.

[2] In Behn-Eschenburg's paper the difference between σ for three-phase and single-phase motors is not dealt with, but a reflection will make it clear that σ is larger for a single-phase winding than for a three-phase winding.

for a the expression $\frac{a}{1+\sigma^k a^2}$. Thus the corrected formula for the power-factor is

$$\cos\phi = \frac{1}{\sqrt{1+\left(\frac{f[1+\sigma_k a^2]}{ab}\right)^2}}, \text{ in which } \sigma_k = \frac{4{\cdot}5}{H^2} + \frac{6\delta}{l}.^1$$

This formula takes account of the leakages of the field flux and armature flux but not of the motor losses. By taking the latter also into consideration, the power-factor will come out from about $\frac{1}{2}$ to $1\frac{1}{2}$ per cent. higher than calculated from the above formula. This small increase may be left to the estimation of dynamo designers.

Turning to the question of *commutation*, it has been shown that—contrary to what happens in d.c. motors—sparking is not worst at full load but at starting at full voltage; for the starting current is considerably larger than full-load current, and the alternating field induces in a short-circuited coil an E.M.F (in volts) of

$$E_w = 4{\cdot}44 \times \text{freq. of supply} \times \text{turns per seg.} \times \text{flux from one pole} \times 10^{-8}.$$

Only small motors are started with the full voltage applied at the terminals. For medium-sized and for large motors the supply voltage is stepped-down at starting. To arrive at a just conclusion, the question of sparking at starting ought to be considered in conjunction with the starting torque the motor is required to develop. If, for example, the motor is specified to exert at starting 4 times the full-load

[1] This formula relates, strictly speaking, to motors only having a distributed stator winding and a constant air-gap all around the armature; it will be shown later on in an example, however, that it also applies to motors with well-defined poles.

torque, the voltage at starting will have to be reduced to such an extent that the starting current is about $2\frac{1}{2}$ times the full-load current. The torque would increase with the square of the current if the iron remained constantly unsaturated; but since this is not the case, more than double the full-load current has to flow through the motor to give the requisite starting torque. Generally speaking, the current will have to be increased from 2·2 to 2·5 times the normal current, and the field flux will have increased to from 1·6 to 1·8 times what it is at full-load. The transformer voltage induced in the short-circuited coil increases in proportion with this field flux.

At starting, the sparking-voltage is

$$E_f = 4{\cdot}4 \times \text{freq. of supply} \times \text{turns per seg.} \times \text{field flux at start.} \times 10^{-8}.$$

If the permissible limit of the sparking voltage, E_f, has been fixed, and if the frequency of supply is given, we get at once from this equation the limiting value of the product (field flux at starting × turns per segment). Since the field flux at starting is fixed for a given starting torque, it is seen that the product (turns per segment × field flux at normal load) is also a fixed quantity, *i.e.*

$$z\mathrm{M} = \frac{E_f\, 10^8}{4{\cdot}4\ a \times \text{frequency}},$$

in which z = turns per segment,

M = field flux at normal load, and

a = ratio of field flux at starting to field flux at normal load.

Taking as an example $a = 2$ and $E_f = 8$, it follows that at a frequency of supply of 12·5, 25, and 50 ~

per second, the product zM must not exceed the values 8×10^6, 4×10^6, and 2×10^6 respectively. The lower the product zM can be kept below these figures the better is the commutation at starting. For a frequency of 50 $\sim$ per second zM must be kept very low to ensure satisfactory commutation at starting, and such low figures are only made possible

(1) By adopting a large number of poles, or

(2) By choosing a small number of turns per segment.

The latter means is only profitable if the armature voltage is low, and it thus follows that the higher the supply frequency the lower ought to be the armature voltage employed.

The effect produced by varying the number of poles will be elucidated by an example:—Assume that a 50-H.P. motor is to be designed to run at a normal speed of 750 revolutions per minute, the frequency of supply being 25 $\sim$ per second. A d.c. motor of the same output and speed would undoubtedly be designed with 4 poles. If our single-phase motor had also four poles, we would get $b=\frac{N'}{N}=1$. But it will be better, with regard both to the commutation and to the power-factor, to design our motor with 6 or 8 poles, corresponding to a $b=1{\cdot}5$ or 2. It is not advisable, however, to have too many poles, since the advantages gained thereby do not keep step with the increase in manufacturing costs. With an increasing number of poles the pole-arc—and, consequently, H also—have to be reduced, and this means that $\sigma_k=\frac{4{\cdot}5}{H^2}+\frac{6\delta}{l}$ increases.

If the diameter of the armature is kept constant while the number of poles is augmented, the expression $\frac{4{\cdot}5}{H^2}$ increases with the square of the number of poles. If, on the other hand, the pole-arc is kept constant, *i.e.* if the armature is increased in diameter and reduced in length, then the expression $\frac{6\delta}{l}$ increases. By increasing the number of poles $a = \frac{\text{armature turns}}{\text{field turns}}$ changes also, unless the armature diameter be increased in the same ratio as the number of poles. This, however, would lead to large diameters and very short lengths. With a given air-gap, the field turns increase almost proportionally with the number of poles, since it is not of advantage to reduce the gap-density. But the armature–A.T. certainly do not grow in the same ratio; they remain nearly proportional to the diameter, and, as already explained, it would not be economical to increase the diameter in the same ratio as the number of poles. The increased number of current-circuits in the armature (in the case of a lap-wound armature[1]) is also a drawback and necessitates a lower armature voltage in order to keep the number of turns per segment the same as when fewer poles are employed. If the armature voltage is fixed by other considerations, however and if it is necessary for this reason to employ a larger number of turns per segment when increasing

[1] A wave winding would be equivalent, as regards commutation, to a lap winding having a correspondingly larger number of turns per segment.

the number of poles, then it is obvious that nothing can be gained in the way of commutation by adopting the larger number of poles. It is thus seen that —quite apart from questions of cost—it is by no means always of advantage to increase the number of poles.

In our 50-H.P. motor, for instance, ten poles would undoubtedly be too many, and it remains to be decided by a careful comparative calculation whether 6 or 8 poles would give the better results.

In the case of a single-phase series motor a reduction in the supply frequency is always of advantage, and this constitutes one of the chief characteristic features of this type of motor and one which distinguishes it, as we shall see later, from the two other representatives of single-phase motors.

If a single-phase series motor, which has been designed for 25 $\sim$ per second, is run from a supply of 12·5 $\sim$ per second, the commutation at starting is improved in the ratio of 1 to 2, and the power-factor has become higher because b in the equation for the power-factor is inversely proportional to the frequency. If, for example, $\cos\phi = 0{\cdot}8$ at 25 $\sim$ per second, it will become 0·935 at 12·5 $\sim$ per second and 0·984 at 6·25 $\sim$ per second. At a sufficiently low frequency, the difference between a d.c. motor and a single-phase series motor is very small.

When the supply frequency is very low, the commutation at starting becomes of less importance than that at full load. According to previous explanations, the sparking voltage at full load is the geometrical sum of three vectors—viz. the reactance voltage O R (fig. 21), the transformer voltage O W,

and the rotation voltage O A. O A, of course, is in phase with the vector O E in fig. 17. At a given current and speed O R is totally independent of the frequency, O W is proportional to the frequency, while O A slowly increases as the frequency diminishes. O C (fig. 17) induces an E.M.F. which counterbalances the ohmic drop in the short-circuited compensating coil, and O C is evidently proportional to the resistance of this coil and inversely proportional to the frequency. If O C is large compared with C D and D E, then O A is approximately inversely proportional to the frequency. If, on the other hand, O C is small compared with C D and D E (*i.e.* if the ohmic resistance is small in comparison with the reactance of the short-circuited compensating coil), then O A (fig. 21) is nearly independent of the frequency.

If the motor is designed for a low frequency, it is thus important to provide a short-circuited compensating coil with a low resistance, otherwise the compensation will be unsatisfactory. In fact, there is no doubt that for very low frequencies a short-circuited compensating coil is to be entirely avoided. In such cases the design of a single-phase series motor becomes quite similar to that of a d.c. series motor. When the frequency is so low that a compensating winding is not necessary, the air-gap density is chosen higher than for high frequencies; at the same time the armature–A.T. and the number of poles are reduced, but each pole becomes bigger of course and is provided with more field–A.T. The length of air-gap loses in importance, and it may be said that such a single-phase series motor

is as good as a d.c. series motor. In many cases it would even be preferred because of the possibility of transforming the voltages up and down, thereby making it possible to use high-tension currents for transmission purposes in conjunction with a low voltage across the armature.

It will be shown later on that these relations as to frequency do not hold good in the repulsion motor and Winter-Eichberg compensated repulsion motor. For this reason the single-phase series motor is always best when the supply frequency is low. True, a low frequency means costly transformers and is unsuitable for electric lighting, but these disadvantages are certainly more than outweighed by the saving in the motors. Transformers, with the exception of the transformer on the car (in the case where motors are used for traction purposes), can be dispensed with, and other advantages are secured by supplying the lighting circuits independently.

The alternators in the power-station would be wound for high voltages (say, from 10,000 to 15,000 volts) and directly coupled to low-speed engines.

The reason why such a high frequency as 25 $\sim$ per second is proposed in connection with single-phase traction is to be sought not so much in the fact that the transformers for such a frequency are cheaper than those designed for fewer cycles per second, or that existing patterns of alternators are unsuitable for a low frequency, but rather in the fact that a frequency of 25 $\sim$ per second suits repulsion motors best and that advocates of other types of single-phase motors feel bound, for commercial reasons, to compete on the same basis.

At comparatively high frequencies the sparking voltage is all-important. An increased sparking voltage may be permitted if resistance is inserted between the armature winding and the commutator segments, as fully explained in a previous chapter. Such a means entails increased losses, and is not as effective as the brush resistance.

In order to keep the sparking voltage per brush within reasonable limits, the brush is made to cover only a few (about two) segments. The normal reactance voltage (O R) could be compensated by replacing the short-circuited compensating winding by a corresponding winding in series with the armature[1] and having more A.T. than the latter. Although such a compensating winding leads to increased cost and a reduced power-factor, it may be employed with advantage when it is found necessary to improve the commutation at full speed rather than at starting.

CHAPTER XIII.

THE REPULSION MOTOR.

Mr Lamme (see the *Journal of the American Institute of Electrical Engineers*, vol. xxi. pp. 61 to 126) was the first to point out that the repulsion motor is essentially a combination of the series motor and the transformer. In view of this, the design of a repulsion motor is more limited in various directions

[1] Such a 'forced' compensating winding is employed by the General Electric Co. of Schenectady, and has also been adopted by the Westinghouse Co. and by the Siemens-Schuckert Co.

than that of a series motor since due regard has now also to be given to the transformer part of the motor. We may remark at once that it is advantageous to employ series motors at speeds beyond synchronous speed, while the best speed for repulsion motors is synchronous speed.

It is easy to design a good series motor for $a=\frac{N'}{N}$ not exceeding 4. In repulsion motors this ratio ought to range from about 0·8 to 1·3. It was shown previously that $\frac{M_y}{M_x}=\frac{N'}{N}$. Hence to obtain a high value of $\frac{N'}{N}$, M_y will either have to be excessively large or M_x unnecessarily small. Both expedients give bad results. A ratio of $\frac{N'}{N}=2$ at full-load is undoubtedly bad. These limiting conditions are of special moment when the frequency of supply is low, since the poles and magnet frame then assume large proportions. In fact, there is quite a wide range of frequencies and sizes for which a satisfactory repulsion motor cannot be made. In this respect the repulsion motor is similar to the induction motor, while the single-phase series motor resembles its d.c. prototype.

The commutation at starting is expressed by the following equation:

$$NzM\,10^{-8}=\frac{E_f}{4\cdot4}.$$

This expression is the same as for a series motor, and in order that the commutation at starting may be satisfactory the product NzM must not exceed a certain value. The latter depends on the kind of

carbon used, etc., and can be determined experimentally without difficulty.

Taking the above equation as a basis, it is of interest to investigate how the constants of the motor vary with the frequency. Let us assume that the maximum sparking voltage permissible is 8 volts. This is a value which may possibly be allowed when very narrow brushes are employed and when resistances are inserted between the armature winding and the commutator segments. Of course, a much smaller figure would be desirable. Let us also assume that the number of turns per segment is reduced to one by adopting a suitable short-circuit current across the armature, and that the flux at starting is $1\frac{1}{2}$ times that at full-load.

Then,
$$N M = \frac{8 \times 10^8}{4{\cdot}4 \times 1{\cdot}5} = 121 \times 10^6.$$

We gather that the maximum flux per pole is fixed as soon as the frequency is given. When the speed is given, the number of poles is fixed and the maximum total flux through the armature (pM) can be calculated on the basis mentioned. It is possible to compute from these relations the limit of capacity for which a satisfactory repulsion motor can be designed. There is, indeed, a definite connection between the capacity, the reactance voltage at full-load, and the flux pM.

It was stated previously that

$$\text{React. volt.} = \frac{4 \times \sqrt{2} \times z \times \text{armat. cur.} \times \text{armat. volt.} \times (g + 0{\cdot}1g')}{p\text{M}}.$$

In our example, therefore,

$$\text{Reactance voltage} = \frac{5{\cdot}6 \times \text{output} \times (g + 0{\cdot}1g')}{p\text{M}}.$$

Now, pM is a given quantity, as explained. The reactance voltage at full-load must not exceed a certain value, say 5 volts. It is thus clear that the output is dependent only on the factor $(g+0{\cdot}1g')$. The latter cannot be made indefinitely small for practical reasons. A very short armature, for instance, would mean a very large diameter. Such a design would not be cheap, and it has also to be kept in mind that the larger the diameter the larger the quantity $0{\cdot}1g'$. We conclude that a satisfactory repulsion motor cannot be built beyond a certain size, the limit depending on the specified speed and supply frequency.

It has already been stated that synchronous speed is the best speed, as regards commutation, for the motor to run at when on full-load. The commutation is then exactly the same as in a d.c. motor of the same dimensions taking the same current and running at the same speed. The reactance voltage is not compensated, and, if due regard is not given to this point when designing the motor, sparking may even occur at synchronous speed. At speeds above that corresponding to synchronism, the commutation rapidly grows worse, as was already indicated by fig. 33.

Not alone the commutation but also the iron losses depend on the ratio $\frac{N'}{N}$, and they disappear when the armature rotates at synchronous speed. In the case of the series motor there is no speed at which the iron losses are zero.

The power-factor of the repulsion motor may be calculated from the formula given in connection with

the series motor. But this does not mean that the power-factor is the same in both cases for the same output, speed, and frequency, because these two types of single-phase motor are so differently designed that the factors in the formula for the power-factor assume different values in each case. As a rule, the power-factor of a repulsion motor will be lower than that of a series motor, as the latter is generally run above synchronism. Since this is especially the case when the frequency of supply is low, it may be said that at low frequencies the power-factor of a series motor is higher than that of a repulsion motor, while at high frequencies the power-factors of both types are approximately the same.

In connection with the paper before the American Institute of Electrical Engineers, referred to above, Mr P. Steinmetz contended that the repulsion motor had an advantage over the series motor inasmuch as the current in the former's short-circuited path had the lead of the primary current. Mr Lamme, on the other hand, suggested that the repulsion motor must be inferior to the series motor because in the former there was the additional leakage of a transformer. Neither contention bears investigation. The lead of the short-circuit current gives rise to more disadvantages than otherwise. It is true that the fact of the rotor current leading the stator current forms the essential difference (as far as the diagram is concerned) between the ordinary induction motor and the repulsion motor. It is also true that the power-factor is improved by this phenomenon, but, on the other hand, it conduces to a lower torque at the same losses and to a worse commutation.

The author does not contend that the series motor is better in this respect, for the current in the short-circuited compensating winding also leads the primary current. In many cases the current in the compensating winding even leads the terminal voltage. This can more easily be achieved in a repulsion motor than in a series motor; but it is never of advantage, as already stated.

With regard to Mr Lamme's remarks, it must be noted that the same leakage which occurs in repulsion motors also occurs in series motors, for the short-circuited compensating coil is nothing else but a transformer winding. And it is well known that the leakage of a transformer which is short-circuited is equal to or even higher than that of a transformer at full-load, assuming the currents to be the same in both cases.

While our conclusions, so far, were not altogether favourable to the repulsion motor, this type of motor nevertheless possesses some characteristic features which render it far superior to the series motor in certain conditions. One of the chief advantages is that the field current and the armature current and the terminal voltage and the armature voltage are quite independent of one another. Thus the designer may adopt any armature voltage he may think fit. This freedom of choice is of special advantage in connection with small stationary motors (such as are used for cranes, lifts, compressors, and the like) which are fed directly from, say, a 500-volt supply. In such cases an armature voltage of from 400 to 500 volts would be objectionable. By employing a lower voltage for the armature,

however, the turns per segment can be reduced to one or two, and this again leads to such a decrease in the sparking voltage that repulsion motors can be designed for 50 or sometimes even 100 ~ per second having a satisfactory commutation although a low efficiency.

The series motor could not compete with the repulsion motor in such conditions, and if the size of the motor be small, even a continuous-current motor might be inferior to a repulsion motor. The superiority of the repulsion motor is still more pronounced when the supply voltage exceeds 500 volts. For large motors this advantage of the repulsion type disappears, since it is then possible to design a series motor also with only one turn per segment. Another good feature of the repulsion motor is the absence of the tendency to flashing-over at the commutator. It has been stated that such a tendency does also not exist in series motors; but it is difficult to see how this can be so, and it is probable that this tendency to flash-over exists in series motors also, although perhaps not in the same degree as in d.c. motors. The fact that the speed of a repulsion motor may be varied by simply shifting the brushes constitutes a further advantage of this type, especially when used as a stationary motor.

In fig. 26 the repulsion motor was shown as having two distinct stator windings. Actually, the motor is provided with one stator winding only, forming a certain angle a with the line connecting the brushes, and the two windings in the figure merely represent the components in the direction of the x-axis and the y-axis respectively. If, for example,

the stator winding consists of n turns, then $n \times \sin a$ is the number of turns producing the field flux, while $n \times \cos a$ turns produce the transformer flux in the direction of the y-axis. Thus the field flux is altered by shifting the brushes. For the repulsion motor we have

$$a = \frac{\text{Stator turns in the direction of the } y\text{-axis}}{\text{Stator turns in the direction of the } x\text{-axis}} = \text{cotan } a.$$

For traction motors, of course, it is advisable to regulate the speed by varying the terminal voltage by means of a transformer.

It was shown in a previous chapter that the short-circuit current in the coils undergoing commutation increases the power-factor at low speeds but reduces it when the speed increases beyond synchronous speed. This has to be remembered when calculating the power-factor, for the formula is only strictly correct when the armature rotates at synchronous speed. At speeds higher than this the effect on the power-factor is particularly pronounced, because the field flux diminishes, as a rule, while the short-circuit current in the coils under the brushes increases rapidly. If a motor, therefore, which had been designed for synchronous speed were run at double this speed, its power-factor would be found to have decreased considerably.

If, on the other hand, the motor were run below synchronous speed, the increase of the power-factor would be insignificant, because the M.M.F. of the field winding is then generally very large and the percentage change in the magnitude and phase of the field flux is, consequently, small.

In the series motor the influence of the coils undergoing commutation is always in the direction of improving the power-factor, and this is a feature in favour of this type.

CHAPTER XIV.

THE COMPENSATED REPULSION MOTOR.

THIS motor has many characteristics in common with the repulsion motor. Its best speed is synchronous speed, and when run much beyond this the same objectionable phenomena appear as in the repulsion motor—viz. a high saturation in the direction of the transformer-axis, a low flux density in the x-axis, and a high sparking voltage.

Since the armature voltage is no longer independent of the stator voltage, it would seem that this type of motor is hardly suitable for small stationary motors (say, for an output less than 10 or 15 H.P.) as an additional transformer would involve an unwarranted expense. The number of sets of brushes is doubled, at least when a lap winding is employed; hence the distance between adjacent sets is halved if the commutator is to remain the same as in the corresponding repulsion motor. But in practice a larger commutator is a necessity in order to dissipate the increased friction losses and C^2R losses under the brushes, this increase amounting to from 20 to 50 per cent. The exact percentage increase depends largely on the 'field'

brushes (the brushes in the x-axis) which, as a rule, may be very much smaller than the short-circuited brushes (the brushes in the y-axis), because not only is the "field" current smaller than the short-circuit current but also because the losses under these brushes are comparatively small. This is clear when it is remembered that no E.M.F. is induced in the coils short-circuited by the 'field' brushes, the only loss in these coils being due to the reactance voltage caused by the commutation of the 'field' current. Besides, this reactance voltage can be compensated by a simple contrivance which, if provided, not only improves the commutation but also reduces the losses considerably.

Against these drawbacks, the compensated repulsion motor has a high power-factor. If the latter, however, is sacrificed to some extent, then many advantages in other directions may be gained. A small air-gap is not as essential as in the repulsion and series motor, and this is a valuable feature for traction purposes. The larger air-gap leads to other advantages, viz. deeper slots on the stator and increased width of slot.

For high frequencies or low speeds the compensated repulsion motor is far better than either of the two other types. In the series motor both the commutation and the power-factor grow worse with an increasing frequency; in a repulsion motor the power-factor diminishes as the frequency increases, its commutation being independent of the frequency. But in the compensated repulsion motor not only is the commutation independent of the frequency (as it is of the repulsion type) but its power-factor is

also independent thereof because of the compensating winding.

The compensation is also the reason why this kind of motor is especially suitable for low speeds. At these low speeds the number of poles required becomes large both for the compensated repulsion motor and the repulsion motor, and the power-factor of the latter type of motor then becomes very bad.

It would certainly be possible to design a series motor for the same low speed with fewer poles, but in this case a low armature voltage would have to be adopted and the power-factor and the commutation would not be good. In designing a low-speed compensated repulsion motor, on the other hand, little attention need be paid to the power-factor (because this always comes out fairly satisfactory on account of the compensation) so that more care can be bestowed on those factors which affect the efficiency and the commutation.

Hence, in comparing these three types of motor due consideration must not only be given to the specified conditions of any particular case but even more to those factors which the designer is at liberty to settle.

When the frequency is as high as, say, 60 $\sim$ per second, the compensated repulsion motor alone deserves serious consideration. This is undoubtedly true for traction motors, but even for stationary purposes it is highly probable that the repulsion motor would be discarded in favour of the compensated repulsion motor. The series motor stands a poor chance at such frequencies.

For very low frequencies, say, from 8 to 16 ~ per second, the series motor is by far the best. Sparking at starting may be almost entirely prevented, even when starting with a large torque, and in all but extreme conditions the power-factor will be as high as in the best modern three-phase motor.

If the frequency is left to the designer's choice, it can only be decided after a careful consideration of all *pros* and *cons* of the particular case whether a series motor and a low frequency or a compensated repulsion motor and a high frequency is preferable. The kind of prime movers (steam turbines, low-speed steam-engines, gas-engines, and so forth) existing or to be employed will also influence the decision one way or the other.

CHAPTER XV.

THE CALCULATION OF A SERIES MOTOR, REPULSION MOTOR, AND COMPENSATED REPULSION MOTOR ON THE BASIS OF EQUAL COST.

In the following chapter we propose to calculate one motor of each type in order to show how the conclusions arrived at in the earlier part of the book may be employed in practice. All questions concerning commutation will be fully dealt with, and the temperature rises will be checked roughly.

The Calculation of a Series Motor (60 H.P.)

Let us assume that our 60-H.P. motor is to be fed from a 25 ~ supply, and is to run at full-load at 500 revolutions per minute. We adopt the following

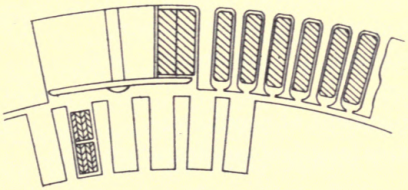

Fig. 46.—Part-section through the 60 H.P. single-phase series motor.

constants and dimensions for the motor, and check backwards to see how the motor behaves. Fig. 46 represents a section through part of the motor.

Number of poles	8

Armature Core.

External diameter of armature,	60	cm.
Internal „ „ stampings,	40	„
Number of slots, ...	96	
Pole-pitch, ...	23·5	cm.

Dimensions of the slot, . . .	$3{\cdot}5 \times 1{\cdot}2$	cm.
Minimum width of tooth, . .	0·53	„
Length between end plates, . .	25	„
Number of ventilating ducts, .	4	
Width „ „ .	1	cm.
Effective length of armature, .	18·9	„

ARMATURE COPPER.

Number of conductors per slot, .	8	
Dimensions of conductor, bare,	$1{\cdot}3 \times 0{\cdot}2$	cm.
Dimensions of conductor, insulated, ...	$1{\cdot}35 \times 0{\cdot}25$	„
Arrangement of conductors, . .	4×2	
Effective length of one armature turn,	38	cm.
Free „ „ „	70	„
Average „ „ „	108	„
Number of armature turns, . .	384	
Number of current circuits in the armature, ...	8	
Number of turns in series between adjacent brushes, . . .	48	

We will adopt a flux density of 18,000 lines per sq. cm. at the root of the teeth at full-load. If we make the pole-arc = 70% of the pole-pitch and assume that the pole flux spreads out 10%, it is found that the section of the teeth (measured at the root) through which the flux from one pole passes is equal to

$$12 \times 0{\cdot}7 \times 1{\cdot}1 \times 0{\cdot}53 \times 18{\cdot}9 = 92{\cdot}5 \text{ sq. cm.}$$

Hence the flux from one pole at full-load :

$$M = 1{\cdot}66 \times 10^6 \text{ lines of force.}$$

We are now able to investigate the commutation at starting. If at this moment the field flux were the same as at full-load, the E.M.F. induced in a coil of one turn short-circuited by the brushes would be

$$E = 4{\cdot}44 \times 25 \times 1{\cdot}66 \times 10^{-2} = 1{\cdot}84 \text{ volts.}$$

This value corresponds to a starting torque equal to full-load torque, and is so low that we may predict with confidence that our motor can start sparklessly with twice or three times the normal torque. In fact, it is easily seen that the motor would start well if the full voltage were applied at its terminals. This is so, because even the greatest rush of current would hardly increase the tooth-density of 18,000 lines per sq. cm. by more than 50 to 70%. Hence the induced voltage E can in no circumstance exceed about 3 volts, and this is so low that sparkless commutation at starting can be obtained with high resistance carbon brushes alone, no resistances between winding and commutator segments being necessary. But such a rush of current would be objectionable for other reasons, and the voltage would on this account be stepped-down at starting.

If twice the normal torque be required at starting, we may estimate, as a first approximation, that the current will have to be 55% above its normal value, the corresponding field flux being 30% greater than at normal load ($1{\cdot}55 \times 1{\cdot}30 = 2$). Thus the induced voltage in a commutating coil at starting comes to 2·38 volts. Such a low value permits the use of a brush covering 3 segments. Since our E corresponds to one turn per segment, our commutator

must obviously have 384 segments. This means a rather large commutator. For the latter we choose:

Diameter of commutator, .	50 cm.
Number of segments, . .	384
Width of segment plus insulation, ...	0·41 cm.
Width of segment alone, .	0·33 „
Width of brush $3 \times 0{\cdot}41 =$	1·2 cm. nearly.

Before determining the length of our commutator let us check the commutation at full-load, because —owing to the high flux-density employed at this load—it may be found to be worse than at starting.

Let J_a = Normal armature current, and
E_a = Armature voltage due to rotation at normal load.

Then, according to the formula on p. 41,

$$\text{Reactance volt. at full load} = \frac{4 \times \sqrt{2}\ J_a \times E_a \times (g + 0{\cdot}1g')}{4 \times \text{flux per pole}}$$

$$= \frac{\sqrt{2}\ J_a \times E_a (38 + 0{\cdot}1 \times 70)}{1{\cdot}66 \times 10^6} = J_a \times E_a \times 38{\cdot}2 \times 10^{-6}.$$

But $E_a = 4 \times 48 \times N \times \text{effective flux per pole} \times 10^{-8}$

$$= 4 \times 48 \times 33{\cdot}3 \times \sqrt{\tfrac{1}{2}} \times 1{\cdot}66 \times 10^{-2} = 75 \text{ volts.}$$

If the current density in the armature copper be 300 amperes per sq. cm., we get

$$J_a = 8 \times 0{\cdot}26 \times 300 = 620 \text{ amperes.}$$

Hence E_r (at full-load) $= 620 \times 75 \times 38{\cdot}2 \times 10^{-6} = 1{\cdot}78$ volts.

The voltages 1·78 and 1·84 are in quadrature, and we get for the sparking voltage at full-load:

$$\sqrt{1{\cdot}78^2+1{\cdot}84^2}=2{\cdot}55 \text{ volts.}$$

Thus the commutation is also satisfactory at full-load. The full-load capacity of the armature is

$$620 \text{ amperes} \times 75 \text{ volts} = 46{\cdot}5 \text{ kw. (60 H.P.).}$$

If we allow a nominal current density under the brush of 6 amperes per sq. cm., we get:

Number of sets of brushes, . .	8
Number of brushes per set, . .	6
Section of one brush, . .	$1{\cdot}2 \times 3{\cdot}6$ cm.
Effective length of commutator, .	26 „
Total length of commutator, .	30 „

The question now arises whether the commutator is large enough to dissipate the heat generated. Assuming a pressure of the brush on the commutator of 0·1 kg. per sq. cm. of contact area and a coefficient of friction of 0·3, the friction losses come to 870 watts. The C^2R losses under the brushes depend on the quality of carbon used and on the sparking voltage. Now, our present case was dealt with very fully from this point of view on p. 23, where it was found that for an assumed contact resistance of 0·3 ohms per sq. cm. contact area the C^2R losses amounted to 5400 watts. The formula for this loss was deduced for a continuous-current motor in which the sparking voltage is in phase with the current.

In an alternating-current motor this is no longer the case, but it can be shown that the losses are independent of the phase difference between sparking voltage and current, *i.e.* the formula quoted

is correct for alternating current as well as for continuous current.

The maximum C^2R loss per sq. cm. occurs at the end of the brush, and we find from the table on p. 23 that this maximum loss per sq. cm. is 105 watts.

In calculating the sparking voltage, we entirely neglected the voltage O A in the commutating coil due to rotation. This must now be done, but before proceeding to carry out the calculation we shall first have to settle the dimensions of the field winding and of the compensating coil. We will fix the following dimensions for the stator:—

Air-gap,	0·3 cm.
Inside diameter of stator stampings,	60·6 „
Pole-arc / pole-pitch,	0·7
Pole-pitch,	23·7 cm
Pole-arc,	16·6 „

We provide each of the 8 poles with 12 semi-closed slots, 4 cm. deep, 0·9 cm. wide, and with an opening at the air-gap of 0·2 cm. (see fig. 46). These slots contain the short-circuited compensating winding, the stranded conductor of which shall have a section of $3·5 \times 0·7$ cm., the copper section being 1·95 sq. cm. The stator teeth have a minimum peripheral width of 0·445 cm. and a maximum width of 0·57 cm.

Calculation of the Field Winding.

Section of armature iron,	246 sq. cm.
„ „ teeth (at root),	92·5 „
„ air-gap,	290 „
„ stator teeth,	109 „
„ „ iron,	236 „

Flux density in armat. core,	6700	lines per sq. cm.
„ „ „ teeth,	18,000	„
„ „ air-gap,	5,750	„
„ „ stator core,	7,000	„
„ „ „ teeth,	15,200	„

We estimate that it requires 1750 field–A.T. per pole to produce the normal flux. The effective A.T. per pole are $\frac{1750}{\sqrt{2}} = 1240$ A.T. Two field turns are thus required per pole (the current is 620 amperes), and we choose a stranded conductor, 3·0 cm. × 0·8 cm. bare and 3·1 cm. × 0·9 insulated, and having a copper section of 1·92 sq. cm.

Each of the 8 short-circuited compensating coils has a resistance of 0·000665 ohms, and the resistance of the field winding is 0·000035 ohms per pole. The 8 field coils are connected in series, the total field resistance being $8 \times 0{\cdot}000035 = 0{\cdot}00428$ ohms. The C^2R loss in the field winding is 320 watts, while the C^2R loss in the short-circuited compensating coils comes to

$$500^2 \times 8 \times 0{\cdot}000665 = 1330 \text{ watts.}$$

The current (500 amperes) in the compensating coils has been computed on the assumption that the A.T. of these coils amount to 80 per cent. of the armature A.T. While the armature winding is uniformly distributed around the armature, the compensating coils only take up from about 60 to 65 per cent. of the periphery, being for this reason more effective. Besides, about 30 per cent. of the armature windings are more or less ineffective owing to

the recesses in the stator to accommodate the field winding. At these places the reluctance is very great, comparatively speaking.

In order to arrive at the magnitude of the flux in the direction of the brushes, we make the assumption that the armature A.T. between the pole-tips (*i.e.* 30 per cent. of the total armature A.T. per pole) have to produce a flux across an air-gap of 0·3 cm. + 4 cm. = 4·3 cm., the flux being in phase with the current. This assumption leads to practically correct results as long as the ordinary air-gap is small compared with the dimensions of the recess. But this assumption would be entirely misleading were, for instance, the laminated stator to extend right around the armature without any recesses, because the flux in the direction of the brushes would then form a considerable percentage of the total flux and would affect the short-circuit current of the compensating coils correspondingly.

According to above, the flux density in the direction of the brushes will be $1{\cdot}25 \times 1100/4{\cdot}3 = 320$ lines per sq. cm., and the E.M.F. induced in a short-circuited armature coil by rotating through this flux is

$$= 1570 \times 42 \times 320 \times 10^{-8} = 0{\cdot}2 \text{ volts.}$$

This voltage is obviously in phase with the current and with the reactance voltage and has to be added to the latter, increasing its value from 1·78 volts to 2·0 volts, very nearly.

The armature A.T. beneath each pole are compensated by the A.T. of the corresponding compensating coil, but the geometrical sum of these two

sets of A.T. is not zero but has a value sufficient to induce in the short-circuited compensating coil an E.M.F. equal to the ohmic drop in voltage in this coil. These resultant A.T. are much smaller, however, than the armature–A.T. between the pole-tips.

Since the reactance voltage itself is only a component of the sparking voltage, we may neglect this small additional voltage (0·2 volts), at least in the present case. In other cases it will be sufficient to estimate it. Hence we need make no corrections backwards with regard to the commutator losses, the values of which we repeat:

Friction losses, . . .	870 watts.
C^2R „ . . .	5400 „
Total commutator losses, .	6270 „

Cylindrical surface of commutator $= 50 \times \pi \times 30 =$ 4700 sq. cm. Thus 1·31[1] watts must be radiated from every sq. cm. Such a value corresponds to a final temperature rise of from 60° to 70° C., and if this is deemed too high, the commutator must be lengthened and the brushes reduced in thickness.

The *power-factor* is calculated from the formula given on p. 98.

Number of slots of the compensating winding per pole (H_1), . . .	$=12$
Number of slots of the armature winding per pole, . . .	$=12$
Pole-arc / pole-pitch, . . .	$= 0{\cdot}7$

[1] A much smaller figure would have been found if the commutator losses had been calculated in the manner usually adopted in practice; this has to be kept in mind when computing the temperature rise from the watts per sq. cm.

$0{\cdot}7 \times 12$ (H_2), ... $= 8{\cdot}4$

$H = \frac{H_1 + H_2}{2}$, ... $= 10{\cdot}2$

Air-gap (δ), ... $= 0{\cdot}3$ cm.

Length of armature (l), . . . $= 25$ „

Dispersion coefficient $\sigma_k = \frac{4{\cdot}5}{10{\cdot}2^2} + \frac{6 \times 0{\cdot}3}{25} = 0{\cdot}115$.

Neglecting the coefficient f (see p. 96), we get

$$a = \frac{\text{armature A.T.}}{\text{field A.T.}} = 3{\cdot}0, \quad b = \frac{N}{N'} = 1{\cdot}33;$$

hence $$\left(\frac{1 + \sigma_k a^2}{ab}\right)^2 = \left(\frac{2{\cdot}0}{4}\right)^2 = 0{\cdot}25,$$

and power-factor $= 1/\sqrt{1{\cdot}25} = 0{\cdot}895$.

It is of interest to check this value by calculating $\cos\phi$ from the diagram of the series motor. The voltage generated in the armature by rotation is 75 volts; the voltage drop in the armature, under the brushes and in the stator winding, is altogether about 14 volts, the watt-component of the voltage thus being 89 volts.

The wattless component is the sum of the E.M.F. induced in the field winding and the E.M.F.'s induced in the armature and compensating windings by the leakage flux. The first-named voltage is

$$4{\cdot}44 \times 25 \times 1{\cdot}66 \times 16 \times 10^{-2} = 29{\cdot}5 \text{ volts},$$

the leakage from the poles being neglected. There are 3750 armature–A.T. per pole, of which only 2630 ($= 0{\cdot}7 \times 3750$) are underneath the pole. All these 2630 A.T. are compensated by the compensating winding except $\sigma_k \times 2630 = 300$ A.T., which induce

an E.M.F. $= 29{\cdot}5 \times 300/1240 = 7{\cdot}1$ volts—assuming the flux produced by these 300 A.T. to be interlinked with 16 turns (*i.e.* the number of field turns). But there are 48 turns on the armature in series between the brushes and only about 60 per cent. of them are interlinked with the leakage flux. It is true that 70 per cent. of the armature turns are beneath the poles, but it is obvious that less than this number is embraced by the leakage flux, and the assumed 60 per cent. will be nearer the mark. Hence the self-induction in the armature and short-circuited winding together amounts to

$$7{\cdot}1 \times 0{\cdot}6 \times 48/16 = 12{\cdot}8 \text{ volts},$$

and the total E.M.F. of self-induction is

$$29{\cdot}5 + 12{\cdot}8 = 42{\cdot}3 \text{ volts}.$$

Therefore

$$E = \sqrt{89^2 + 42{\cdot}3^2} = 99 \text{ volts and } \cos\phi = 89/99 = 0{\cdot}9.$$

The agreement between the values of $\cos\phi$ calculated in two different ways is thus very fair.

The *reaction of the commutating coils* upon the power-factor can be taken into account as follows:— It was shown on p. 46 that the M.M.F. of the coils undergoing commutation is given by the expression

$$\text{M.M.F.} = 0{\cdot}083\ n^2\ z\ E_r/W.$$

In our particular case $n = 3$, $E_r = 2{\cdot}55$ volts,

W = resist. of one set of brushes $= 0{\cdot}3/26 = 0{\cdot}0115$ ohms, and z = turns per segment $= 1$.

Therefore $\text{M.M.F} = 0{\cdot}083 \times 9 \times 2{\cdot}55/0{\cdot}0115 = 160$ A.T.

We now find that, referring to fig. 47,

$$O\,B = 1240 \text{ A.T. per pole,}$$
$$O\,R = 1{\cdot}78 \text{ volts,}$$
$$B\,B' = 160 \text{ A.T. per pole,}$$
$$O\,W = 1{\cdot}84 \text{ volts, and}$$
$$O\,F = \sqrt{1{\cdot}78^2 + 1{\cdot}84^2} = 2{\cdot}55.$$

B B′ is parallel to O F, and we find by calculation O B′ = 1355 and angle B′O B = 5 degrees, nearly.

It is therefore seen that the effective field flux is increased by the action of the commutating coils by 9 per cent. but lags now 5 degrees behind the current. This results in a diminution of the motor speed of some 6 to 8 per cent., and the main current will lag—at this reduced speed—27·5 degrees behind the terminal voltage. Subtracting from this the 5 degrees between O B and O B′, we find that the corrected phase-difference between terminal voltage and main current is 22·5 degrees, corresponding to a power-factor cos $\phi = 0{\cdot}925$. Of course, the above calculation is strictly correct for sine waves only, and the actual value of cos ϕ will be somewhat smaller because such sine waves never obtain in practice. Tables II. and III. contain the losses in, and the weights of, the different parts of our 60-H.P. motor.

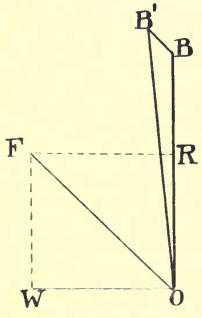

FIG. 47.

TABLE II.—LOSSES (IN WATTS).

Iron losses in the stator,	700
„ „ armature,	800
C^2R losses in the armature,	1920
„ „ field winding,	320
„ „ compensating winding,	1330
Brush friction losses,	870
C^2R losses under the brushes,	5400
Bearing friction,	600
Total losses,	11,940
Output of motor,	44,360
Input,	56,300
Efficiency,	79 %

TABLE III.—WEIGHTS (IN KG.).

Armature copper,	96
Field copper,	60
Copper of compensating winding,	92
Weight of armature stampings (full),	225
„ stator „ „	320
„ commutator copper,	160
Total weight of effective material,	953

It cannot be said that the design is particularly cheap, but attempts to reduce the cost lead to higher sparking voltages. In this respect all depends on the specified conditions as to sparking. While our motor will just operate sparklessly with high-resistance carbon brushes, it would undoubtedly

spark badly if two turns per segment were employed unless special contrivances are adopted. It might be made to run fairly well if, for instance, resistances were inserted between the armature winding and the commutator segments, but such a solution would conduce to an increase in the losses and in the winding space and would lead to a higher temperature rise and to bad ventilation. In any case, resistances can only be decided upon after a very careful comparative calculation of various alternative designs.

In comparison with a d.c. motor of the same size, the above single-phase series motor is characterised by its large commutator. This is chiefly the result of the low terminal voltage, the value of which is

$$\frac{\text{input}}{\text{current} \times \cos\phi} = \frac{56{,}300}{620 \times 0{\cdot}925} = 98 \text{ volts.}$$

For a terminal voltage of, say, 250 volts, however, the commutation would have been very unsatisfactory, and without special devices for reducing sparking it would be practically impossible to arrive at a good design. The design with two turns per segment, referred to above, would have allowed a terminal voltage of 196 volts to be adopted.

It is interesting to note that the air-gap of our motor is large, the power-factor being high none the less. A three-phase motor of the same dimensions would have a power-factor of about 0·80 or 0·85, and its no-load current would be large—about 40 per cent. of the full-load current.

If the frequency of supply had been 12·5 $\sim$ per second instead of 25 $\sim$ per second, a much better

motor might have been designed. A 6-pole motor would then have been chosen; the diameter of the armature would have been reduced, the length being slightly increased. A higher terminal voltage could have been adopted, and this would have conduced to a smaller commutator and to a decrease of the commutator losses. The efficiency, power-factor, temperature rise, and commutation would all have been improved. The effected saving in material would not only have meant lower cost but also less bulk, a great consideration in traction motors.

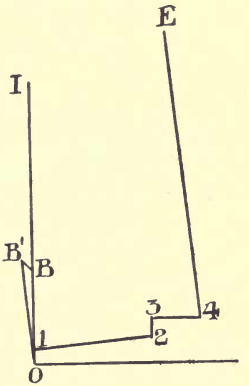

FIG. 48.—The terminal voltage and its components at full-load.

For a still lower frequency of, say, 5 to 8 cycles per second, the design would become similar to that of a d.c. motor. The compensating winding would then be dispensed with, higher flux-densities would be adopted, and the armature could be reduced still more. However, the number of poles would still be larger than in the corresponding d.c. motor; moreover, the ratio of pole-arc to pole-pitch would have to be very small in order to reduce, as far as possible, the armature cross A.T. and the armature flux in the direction of the brushes. This would be necessary from the commutation point of view.

Fig. 48 represents the diagram of our 60-H.P. motor at full-load, the general diagram for a constant terminal voltage being given in fig. 49. The

latter is correct for smaller loads, but holds good no longer for over-loads owing to saturation.

The saturation of the iron parts has the effect of increasing the short-circuit current of the motor beyond the value indicated in fig. 49. This means

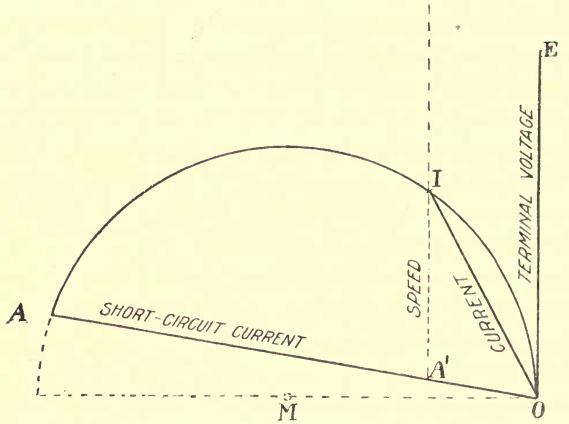

FIG. 49.—The diagram of the 60-H.P. series motor for a constant terminal voltage.

that the actual maximum torque and the actual maximum capacity of the motor are larger than implied by fig. 49, so that it is permissible to work the motor at full-load near to its apparent maximum capacity.

CALCULATION OF A REPULSION MOTOR.

We adopt the following dimensions and check backwards as before:—

Number of poles,	6
Frequency of supply,	25 ~ per sec.
Speed at full load,	500 revs. per min.

Armature:

Outside diam. of armature, .	50 cm.
Inside „ „ .	25 „
Number of slots (the slots are open), . . .	80
Dimensions of a slot, . .	3·7 × 1·1 cm.
Pole-pitch, ...	26·2 cm.
Minimum width of tooth, .	0·57 „
Length between end-plates, .	36 „
Number of ventilating ducts,	6
Width „ „ .	1 cm.
Effective length, . . .	27 „

Armature copper:

Number of conduct. per slot,	8
Dimens. of conductor (bare),	1·45 × 0·18 cm.
„ „ (insulated),	1·50 × 0·23 „
Arrangement of conductors,	2 × 4
Effective length of one turn,	54 cm.
'Free' length of one turn, .	78 „
Average „ „ .	132 „
Number of armature turns, .	320
„ current circuits, .	6
„ armature turns in series between brushes, .	53⅓

Stator iron:

Air-gap, ...	0·3 cm.
Inside diameter of stampings,	50·6 „
Outside „ „	70 „
Length between end-plates,	36 „
Number of ventilating ducts,	6
Width „ „	1 cm.
Effective length, . . .	27 „

Number of slots, . . .	110
Dimensions of one slot, .	4·0 × 0·9 cm.
Width of slot at air-gap, .	0·4 cm.
Minimum width of tooth, .	0·34 „

Stator winding:

Number of conduct. per slot,	8
Arrangement of conductors, .	2 × 4
Dimens. of conductor (bare),	1·55 × 0·125 cm.
„ „ (insulated),	1·60 × 0·175 „
Number of turns in series, .	220
Average length of a turn, .	153 cm.
Resistance of stator winding,	0·175 ohms.

Commutator:

Diameter of commutator, .	43 cm.
Useful length of „ .	34 „
Number of segments, . .	320
Width of one segment + insulation, ...	0·42 cm.
Width of one segment alone,	0·35 „

Brushes:

Number of sets of brushes, .	6
„ brushes per set, .	6
Dimensions of one brush, .	0·8 × 5·1 cm.

We fix the maximum flux density in the narrowest part of the armature teeth at full-load at 18,500 lines per sq. cm.

The minimum section through the teeth per pole is

$$13{\cdot}3 \times 0{\cdot}57 \times 27 = 205 \text{ sq. cm.}$$

But the flux density varies from point to point along the periphery of the stator. Assuming the distribution of the flux to follow the sine law, the

flux density at two points of the stator, one pole-pitch distant from one another, would be zero, while at a third point, midway between the first two points, the flux density would be $\frac{\pi}{2}$ times the average density. The actual distribution is not sinusoidal, however, (as is fully explained in Appendix I.), and this will be taken into account, in our particular case, by substituting 1·75 for $\pi/2$.

Thus the average flux density is 18,500/1·75 = 10,600.

Field flux per pole = $205 \times 10{,}600 = 2{\cdot}18 \times 10^6$ lines of force.

The transformer voltage induced by this flux in an armature coil undergoing commutation is

$$= 4{\cdot}44 \times 25 \times 2{\cdot}18 \times 10^{-2} = 2{\cdot}42 \text{ volts.}$$

If the starting torque is specified to equal twice the normal torque, we get, approximately,

Sparking voltage at starting = $1{\cdot}3 \times 2{\cdot}42 = 3{\cdot}15$ volts.

In our previous example (60-H.P. series motor) the sparking voltage at starting was found to be 2·38 volts, and we were justified in choosing a brush which covered three segments. If our repulsion motor is to have as good a commutation at starting as the series motor, we shall have to reduce its brush width so that, say, only two segments are covered.

As soon as the armature begins to rotate, a second voltage is induced between the brushes. At synchronous speed this voltage, due to rotation through the field flux, is

$$E_b = 4 \times 25 \times s \times M_x \times 10^{-8} = 5333 \times M_x \times 10^{-8},$$

in which M_x denotes the momentary value of the field flux. E_b pulsates in accordance with M_x and attains its maximum value when M_x is at its maximum (*i.e.* $2{\cdot}18 \times 10^6$ lines). Thus the maximum value of E_b is

$$5333 \times 2{\cdot}18 \times 10^6 \times 10^{-8} = 116 \text{ volts.}$$

If we assume that M_x follows the sine law, as regards time, the effective value of E_b will be

$$E_b = 106/\sqrt{2} = 82 \text{ volts.}$$

On the further assumption that the current density in the armature is the same as in the previous example, the total armature current comes to

$$6 \times 78 = 468 \text{ amperes,}$$

and the electrical input into the armature is

$$82 \times 468 \times \cos \phi_{12} = 38{,}400 \times \cos \phi_{12},$$

in which $\cos \phi_{12}$ is the phase-difference between the field flux and the armature current. This angle ϕ_{12} is very nearly equal to the phase-difference between the stator current and the armature current. ϕ_{12} will be calculated accurately later on; for the present we estimate $\cos \phi_{12} = 0{\cdot}95$. The reactance voltage, at full-load, is

$$\frac{4 \times \sqrt{2} \times 82 \times 468\,(54 + 7{\cdot}8)}{3 \times 2{\cdot}18} = 2{\cdot}04 \text{ volts.}$$

Our motor is so designed that its armature rotates, at full-load, at synchronous speed. It was explained in a previous chapter that at this speed all voltages in the coil undergoing commutation balance each other

with the exception of the reactance voltage pure and simple.[1]

The field A.T. and power-factor are calculated in the following manner :—

Section of armature core, . .	312 sq. cm.
„ „ teeth per pole,	205 „
„ air-gap per pole, . .	707 „
„ stator teeth per pole, .	248 „
„ stator iron, . . .	312 „
Max. flux density in armat. core,	7,000 lines per sq. cm.
„ „ „ teeth, .	18,500 sq. cm.
„ „ air-gap, . .	5,400 „
„ „ stator teeth, .	15,500 „
„ „ „ iron, .	7,000 „
Maximum A.T. required for air-gap,	1,300
„ „ „ iron paths,	250
Total field A.T. per pole (maximum),	1,550
Total field A.T. per pole (effective),	1,100

In order to be able to calculate the necessary number of the transformer A.T., we require to know the value of σ_k :

Number of armature slots per pole $(H_1) = 13{\cdot}3$;

„ stator „ „ $(H_2) = 18{\cdot}3$.

$$H = \frac{13{\cdot}3 + 18{\cdot}3}{2} = 15{\cdot}8.$$

$$\sigma_k = \frac{4{\cdot}5}{15{\cdot}8^2} + \frac{6 \times 0{\cdot}3}{30} = 0{\cdot}018 + 0{\cdot}06 = 0{\cdot}078.$$

[1] Strictly speaking, this is only true when the flux is distributed in a sinusoidal manner around the armature. A complete analysis in this part of the book would have obscured the chief points at issue. A complete investigation will be found in Appendix I.

In the triangle A′B O (fig. 50):

A′B = armature A.T. per pole = $53{\cdot}33 \times 78 = 4160$;

O B = field A.T. per pole $\times \frac{N}{N'}$; at synchronous speed

$$O\,B = 1100 \text{ A.T.}$$

The angle B O A′ = 90 degrees.

The transformer A.T. per pole are

$$\left(1+\frac{\sigma_k}{2}\right) \times O\,A' = 1{\cdot}04\sqrt{4160^2 - 1100^2} = 4180 \text{ A.T.}$$

It has been already pointed out that the transformer coil and the field coil do not exist separately but are combined in one single coil forming an angle α with the assumed transformer coil.

We have $$\tan \alpha = \frac{\text{field A.T.}}{\text{transformer–A.T.}};$$

$$\text{stator A.T.} = \sqrt{(\text{field A.T.})^2 + (\text{transformer A.T.})^2}$$
$$= \sqrt{4180^2 + 1100^2} = 4340 \text{ A.T.}$$

Thus the stator current $= \frac{6 \times 4340}{220} = 118$ amperes.

At synchronous speed the power-factor is

$$\cos \phi = \sqrt{\frac{1}{1+\left(\frac{1+\sigma_k a^2}{a \times b}\right)^2}}.$$

For the repulsion motor as well as for the Winter-Eichberg motor

$$a = \frac{\text{transformer A.T.}}{\text{field A.T.}} = \text{cotan } \alpha.$$

In our case $$a = \frac{4180}{1100} = 3{\cdot}8.$$

Hence $$\cos \phi = 0{\cdot}86.$$

It is of some interest to verify this result by a method which makes use of the clock-face diagram. Fig. 50 gives us, approximately, the phase of the armature current, and we make use of this to construct the exact diagram (fig. 51).

O B (at synchronous speed) = field A.T. per pole = 1100 A.T.

B B′ is perpendicular to the vector of the armature

FIG. 50.

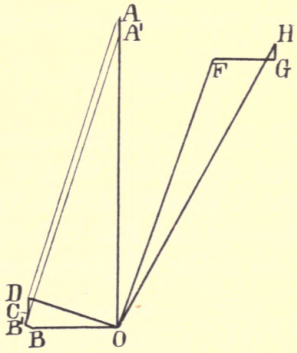

FIG. 51.—Voltage diagram of the 48-H.P. repulsion motor.

current, and is proportional to the drop of voltage in the armature winding and under the brushes.

The armature resistance is $\frac{53{\cdot}3 \times 1{\cdot}32 \times 0{\cdot}02}{6 \times 26} =$ 0·009 ohms; hence, drop of voltage in the armature winding = $468 \times 0{\cdot}009 = 4{\cdot}2$ volts. Provisionally, we estimate the drop under the brushes to be 3·5 volts, so that the total drop of voltage in the secondary circuit comes to

$$4{\cdot}2 + 3{\cdot}5 = 7{\cdot}7 \text{ volts.}$$

It was shown that a voltage of 82 volts is induced in the armature winding by the field flux. The A.T., therefore, which are necessary to produce an E.M.F. which will compensate the drop of 7·7 volts are

$$1100 \times \frac{7{\cdot}7}{82} = 100 \text{ A.T.}$$

$$\text{O A}' = 4020 \text{ and O A} = 4020\left(1 + \frac{\sigma_k}{2}\right) = 4020 \times 1{\cdot}04 = 4180 \text{ A.T.}$$

These two vectors are at right angles to O B. Let

$$\text{A}'\text{C (C being on A}'\text{B}') = \left(1 - \frac{\sigma_k}{2}\right)\text{A}'\text{B}' = 0{\cdot}96\ \text{A}'\text{B}',$$

then D is the point of intersection of a line drawn through A parallel to A′B′ and of a line drawn through C parallel to O A. We measure O D from the diagram and find O D = 1250 A.T. These A.T. give rise in the transformer winding to a voltage

$$82 \times \frac{\text{O D}}{\text{O B}} \times \frac{\text{turns in the transformer winding}}{\text{armat. turns in series betw. the brushes}}$$

$$= 82 \times \frac{1250}{1040} \times \frac{220 \times \cos \alpha}{53{\cdot}33} = 380 \text{ volts.}$$

Let this voltage be represented by O F, perpendicular to O D. F G is the voltage induced in the field winding by the field flux, its value being

$$82 \times \frac{220 \times \sin \alpha}{53{\cdot}33} = 81 \text{ volts.}$$

G H is the voltage drop in the stator and is equal to $0{\cdot}175 \times 118 = 20{\cdot}7$ volts. O H represents the terminal voltage, and it is found from the diagram that O H = 435 volts. Hence $\cos \phi = \cos$ (angle A O H) = 0·88. It is thus seen that the value of $\cos \alpha$ obtained

from the formula (0·86) is 2 per cent. smaller than the actual value, this difference being due to the fact that the losses were neglected in the former case.

Commutation and Commutator Losses.—Assuming again that carbon brushes having a contact resistance of 0·3 ohms per sq. cm. are employed, the distribution of current under the brushes at synchronous speed is approximately as shown in fig. 52. At the right-

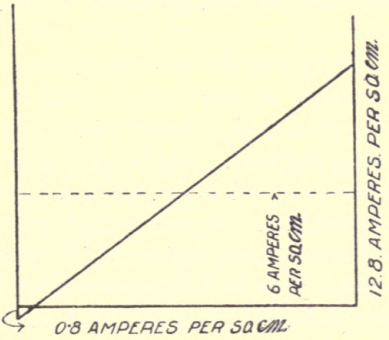

FIG. 52.—Distribution of current under the brushes of the 48-H.P. repulsion motor.

hand side of the brush the current-density is 12·8 amperes per sq. cm. while at the left-hand side it is 0·8 amperes per sq. cm., corresponding to a drop of voltage between brush and commutator of 3·84 volts and 0·24 volts respectively.

At synchronous speed the effective drop of voltage under the brushes is

$$2\times\left(1{\cdot}8+\frac{4{\cdot}1^2}{12\times 1{\cdot}8}\right)=5 \text{ volts.}^1$$

[1] This voltage drop is thus a little larger than that estimated above, but the difference is so small that it is not necessary to repeat the calculations.

We are now able to compute the commutator losses:

C^2R losses under the brushes $= 468 \times 5 = 2340$ watts.
Friction losses (assuming a pressure of 0·1 kg. per sq. cm. contact area and a friction-coefficient of 0·3), . = 515 „
Total commutator losses, . = 2855 „

Since the cylindrical surface of the commutator is 4600 sq. cm., each sq. cm. is called upon to radiate 0·62 watts.

There is hardly any doubt—considering the small reactance voltage of our motor—that it would be preferable to use a carbon brush with a lower contact resistance than assumed. But it must be remembered that synchronous speed is by far the best speed as regards commutation. In practice, motors will usually be required to operate satisfactorily over a wide range of speed; we will therefore investigate the motor as to sparking for a couple of other speeds also, assuming that the terminal voltage remains constant and that the position of the brushes is left unaltered.

Let us first consider the speed at which the stator current is $1\frac{1}{2}$ times full-load current. Although the field A.T. have increased 50 per cent., the actual increase in field flux is much less (about 25 or 30 per cent.) owing to the high saturation—18,500 lines per sq. cm.—in the teeth at normal load. If required, the exact value of the new field flux can be calculated with the aid of the saturation curve. It must be remembered, however, that the distribution of flux along the air-gap does not follow the sine law when the flux densities are high; in such cases the distribution is represented by a curve the top of which

is more or less flat. The actual distribution may be determined by first plotting the saturation curves in the directions of various radii, and then making use of them for calculating the flux densities at the different points in the air-gap corresponding to the various radii.

In the present case an assumed 25 per cent. increase of the flux density in the direction of the x-axis would correspond to an apparent density in the rotor teeth of 23,200 lines per sq. cm. and in the stator teeth of 19,400 lines per sq. cm. The requisite field A.T. would amount to

Air-gap A.T., ...	1630	
A.T. required to force the flux through the teeth, . . .	850	
Total, ...	2480	A.T.

Actually, however, our field–A.T. have only risen to 2320 A.T. ($1550 \times 1{\cdot}5$), so that it is apparent that the flux density in the direction of the x-axis cannot have increased by quite as much as the assumed 25 per cent. We may estimate with some degree of accuracy, however, that the increase of the flux density in the direction indicated amounts to 24 per cent. At other points along the air-gap the increase will certainly be larger, because the original flux densities and the saturation at these points were smaller than that in the direction of the x-axis. Altogether we shall not err greatly by assuming the total field flux to have increased by 30 per cent. Correspondingly, the self-induction in the field winding is 30 per cent. greater than at synchronous speed, and the self-induction in the armature and

transformer windings is 50 per cent. higher than before.

In fig. 51 the self-induction of the field winding is equal to the distance between the point F and the vector O A, while the self-induction in the armature and transformer windings is represented by the vector F G. We have now

Self-induction in the field winding $= 1{\cdot}3 \times 81 = 105$ volts, and self-induction in the armature and transformer windings $= 1{\cdot}5 \times 120 = 180$ volts.

Hence, wattless component of the terminal voltage $= 105 + 180 = 285$ volts, and watt component of the terminal voltage $= \sqrt{435^2 - 285^2} = 332$ volts.

The total ohmic drop, which was 52 volts at synchronous speed, is now $52 \times 1{\cdot}5 = 78$ volts. At synchronous speed the 'rotation-voltage' was 330 volts; now it is $332 - 78 = 254$ volts.

From this we obtain the new speed, which is

$$\frac{254}{330} \times 500 = 0{\cdot}59 \times 500 = 295 \text{ revs. per min.}$$

The power-factor has become $332/435 = 0{\cdot}76$.

The new sparking voltage is calculated as follows: Reactance voltage $= 2{\cdot}04 \times 0{\cdot}59 \times 1{\cdot}5 = 1{\cdot}8$ volts; voltage induced in a coil undergoing commutation by the transformer action of the field flux $= 2{\cdot}42 \times 1{\cdot}3 = 3{\cdot}1$ volts; 'rotation voltage' in the short-circuited coil $= 2{\cdot}42 \times 1{\cdot}3 \times 0{\cdot}59^2 = 1{\cdot}1$ volts. The geometrical sum of 'transformer voltage' and 'rotation voltage' is $3{\cdot}1 - 1{\cdot}1 = 2$ volts.

Since the armature current and, therefore, the

reactance voltage also, are nearly in phase with the stator current at the lower speed, we get

$$\text{Sparking voltage } E_f = \sqrt{1{\cdot}8^2 + 2^2} = 2{\cdot}7 \text{ volts.}$$

This voltage is within the permissible limits.

Let us now investigate how the motor behaves at a speed higher than synchronous speed. We will assume a speed corresponding to a decrease of the normal current to 60 per cent. The flux M_x will have fallen to about 0·65 $M_{x\ \text{synchr.}}$ We have now

$$\text{Self-induction} = 0{\cdot}65 \times 81 + 0{\cdot}6 \times 120 = 124 \text{ volts.}$$

$$\text{Watt-component of terminal voltage less total ohmic drop} = \sqrt{435^2 - 124^2} - 0{\cdot}6 \times 52 = 385 \text{ volts.}$$

$$\text{Speed} = 500 \times \frac{385}{330} \times \frac{1}{0{\cdot}65} = 900 \text{ revs. per min.}$$

O B (fig. 50) has increased in the ratio 385/330, while O A has diminished in the ratio 0·6/1. Hence

$$\cos \phi_{12} = 0{\cdot}896.$$

It is also found that the various voltages in the coil undergoing commutation are as follows :—

Reactance voltage	=	2·4 volts.
'Transformer voltage'	=	1·6 „
'Rotation voltage'	=	4·5 „

The geometrical sum of the 'transformer voltage' and the 'rotation voltage' is $4{\cdot}5 - 1{\cdot}6 = 2{\cdot}9$ volts, in phase with the 'rotation voltage.'

The reactance voltage has a component in phase with the 'rotation voltage' of the value $2{\cdot}4 \times \sin \phi_{12}$, and another component in phase with the stator

current equal to $2{\cdot}4 \times \cos \phi_{12} = 2{\cdot}15$ volts. Hence Sparking voltage $E_f = \sqrt{(2{\cdot}9+1{\cdot}1)^2+2{\cdot}15^2} = 4{\cdot}55$ volts.

It is of some interest to investigate the reaction of the coils undergoing commutation upon the field flux and its effect upon the power-factor.

As previously explained, the A.T. of the commutating coils may be calculated from the formula

$$\text{A.T.} = 0{\cdot}083\ n^2\ E_f\ z/W.$$

In our case $n=2$, $E_f=4{\cdot}55$, $z=1$, and W = contact resistance of all the brushes of one set $= \dfrac{0{\cdot}3}{0{\cdot}8 \times 5{\cdot}1 \times 6}$ $= 0{\cdot}0115$ ohms. Therefore

$$\text{Reaction per pole} = 0{\cdot}083 \times \frac{4 \times 4{\cdot}55}{0{\cdot}0115} = 130 \text{ A.T.}$$

At synchronous speed, the M.M.F. of the field winding is 1100 A.T. per pole. However, the field winding is distributed over the stator, while the coils short-circuited by the brushes must be considered as embracing the *whole* of the flux. In a previous chapter the factor 1·75 was adopted to represent the ratio of the effect of a concentrated winding to the effect of a distributed winding. Taking the same factor, it is found that the equivalent reaction of the commutating coils amounts to $1{\cdot}75 \times 130 = 228$ A.T.

In fig. 53, S S′ = 228 A.T. and O S = 660 A.T. The resultant O S′ = 800 A.T. Angle S O S′ is measured and found to be 14·5 degrees.

The power-factor, as calculated in the usual way, is $417/435 = 0{\cdot}96$; but if the above reaction of the

commutating coils is taken into account, it is seen that the power-factor only comes to $\cos \phi = 0{\cdot}85$.

The losses at full-load (synchronous speed) are:—

Iron loss in the armature,	. .	200 watts.
„ „ stator,	. .	1350 „
C^2R loss in the armature,	. .	1970 „
„ „ stator,	. . .	2440 „
Friction losses of the brushes,	.	510 „
C^2R losses under the brushes,	.	2340 „
Bearing friction,	. . . .	600 „
Total losses,	. . .	9410 „

Output $38{,}000 \times \cos \phi_{12} - (600 + 510 + 200) = 35{,}500$ watts
Input . . . . . $= 44{,}910$ „
Efficiency . . . . . $= 79\,\%$

Compared with the series motor, our repulsion motor does not show up well, for the dimensions of both motors were so chosen that the manufacturing costs came to about the same in each case. While the repulsion motor has a capacity of only 48 H.P., the series motor developed 60 H.P. at full-load. When the repulsion motor is running at synchronous speed its efficiency is the same as that of the series motor. At other speeds, however, the efficiency of the latter is superior to that of the repulsion motor.

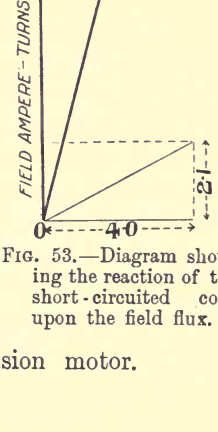

FIG. 53.—Diagram showing the reaction of the short-circuited coils upon the field flux.

Calculation of a Compensated Repulsion Motor

As example let us take the repulsion motor just calculated, converting it to a compensated repulsion motor by simply adding 6 new sets of brushes, each of which conveys $118/3 = 39{\cdot}3$ amperes. In this particular instance the M.M.F. of the field winding (which is now on the armature) is very nearly the same as the M.M.F. of the field winding on the stator in the case of the repulsion motor. This has been accomplished by choosing a suitable transformer-ratio in the repulsion motor dealt with above.

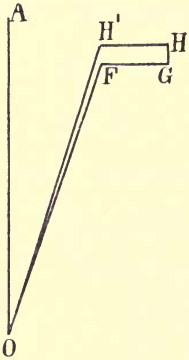

Fig. 54.—Voltage diagram of the compensated repulsion motor.

Each new set of brushes consists of 2 brushes ($0{\cdot}8 \times 5{\cdot}1$ cm.). They occasion a friction loss of 170 watts and a C^2R loss of 150 watts. The commutation under the main brushes remains the same as before. For the same current the terminal voltage has become lower and the power-factor higher.

Fig. 54 is the reproduction of fig. 51 with the addition of a vector to represent the E.M.F. in the field winding. At synchronous speed this E.M.F has exactly the same value as the self-induction in the field winding; hence $H H' = G F$. The power-factor has risen from 0·86 to 0·95, while the terminal voltage has fallen from 435 volts to 405 volts.

Since nothing new has to be taken into account otherwise, the calculations made in connection with

the repulsion motor are directly applicable to our compensated repulsion motor. The main brushes of course are now exactly in the y-axis. Although the power-factor is considerably better than before, the efficiency has suffered. The increase in the commutator losses amounts to 320 watts, the armature losses have increased by 220 watts, while the C^2R losses in the motor windings are 8 per cent. less than before. This decrease is due to the fact that the brushes are now in the centre line of the stator winding. Hence less turns per pole are required on the stator, or, if the number of turns be left unaltered, the terminal voltage will be somewhat increased, the primary current being reduced to some extent at the same time. In our case this reduction will be

$$\left(1-\frac{\text{transformer turns}}{\text{stator turns}}\right)\times 100 = 4 \text{ per cent.}$$

This means 8 per cent. reduction in the C^2R losses, *i.e.* 195 watts, so that the actual increase in the motor losses is $320+220-195=345$ watts. The efficiency drops to 78·2 per cent. in consequence.

All three motors have a lap winding on the armature. A wave winding is out of the question, for a careful investigation shows that—in respect to commutation—a wave winding with one turn per segment is equivalent to a lap winding with p turns per segment, p being the number of pole pairs. A wave-wound armature may be adopted therefore in 4-pole motors in which two turns per segment are permissible, or in 6-pole motors in which three turns per segment are not objected to.

APPENDIX I.

A CONTRIBUTION TO THE EXACT THEORY OF THE REPULSION MOTOR

The ideal winding of a repulsion motor would be that winding for which the variation of both the field flux and the transformer flux along the air-gap followed the sine law. Assuming such a winding to be possible, the E.M.F. in the armature could be calculated as follows:—

Let $B_{x\,max.}$ = maximum field flux density in gap,
$B_{x\,mean}$ = average field flux density in gap,
Q = sectional area of air gap.
Then Field flux $M_x = Q\,B_{x\,mean}$.

At synchronous speed the maximum E.M.F. induced between the short-circuited brushes by rotation is

$$= 4 \times s \times N \times M_x \times 10^{-8},$$

and at any other speed U (corresponding to a frequency of rotation N′) the maximum E.M.F. is

$$= 4\,s\,N'M_x\,10^{-8}.$$

Hence the effective voltage between the short-circuited brushes at a speed U, assuming the flux to vary also in a sinusoidal manner with the time, is

$$E_x = \frac{4}{\sqrt{2}}\,s\,N'M_x\,10^{-8} = 2{\cdot}82\,s\,N'M_x\,10^{-8}.$$

If the transformer flux were linked with *all* the armature turns, it would induce in them an effective E.M.F.

$$= 4{\cdot}44\,s\,N\,M_y\,10^{-8}.$$

But this is not the case because the stator winding is uniformly distributed over the stator, and for this reason

the actual voltage induced in the armature is $\pi/2 = 1{\cdot}57$ times smaller than the value just mentioned.

$$\text{Therefore } E_y = \frac{4{\cdot}44}{\pi/2} s\, N\, M_y\, 10^{-8} = 2{\cdot}82\, s\, N\, M_y\, 10^{-8}.$$

With the exception of the drop of voltage in the armature and under the brushes—which drop we will neglect for the moment—E_x and E_y are the only E.M.F.'s in the armature, and they must necessarily balance each other. Thus, taking for the present no notice of phase relations,

$$N\, M_y = N' M_x.$$

This is the fundamental equation of the repulsion motor.

A winding of the perfect type assumed is distinguished by the following two features :—

(1) The armature iron losses are zero at synchronous speed.

(2) At synchronous speed the sum total of all voltages (except the reactance voltage pure and simple) in the coil short-circuited by the brushes is zero.

These two propositions are proved as follows :—

At synchronous speed $M_x = M_y$, *i.e.* $B_x = B_y$, and the phase-angle between M_x and M_y is 90 degrees. A rotating field is thus created which rotates in the same direction and at the same speed as the armature. It induces no E.M.F.'s in the armature, and gives rise neither to hysteresis nor eddy-current losses.

However, such a perfect winding as that assumed in the foregoing has yet to be invented, and the windings employed in practice conduce to a distribution of flux around the armature differing considerably, no doubt, from a sinusoidal distribution.

For the purpose of investigation we imagine the stator to have two ordinary d.c. windings. Winding I. (fig. 55)

represents the transformer winding, winding II. is the field winding, while III. is the armature winding.

We imagine the stator windings cut open and spread

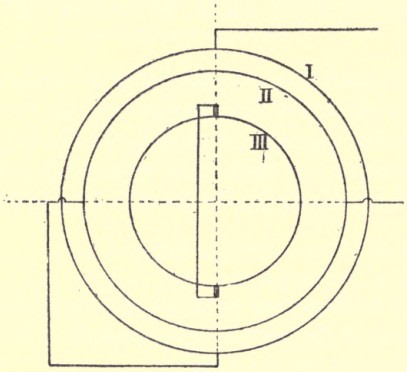

FIG. 55.

out in a plane so that A C (fig. 56) represents the circumference of our armature and any ordinate of the triangular curve represents the value of the M.M.F. of

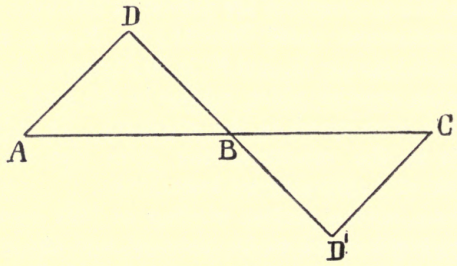

FIG. 56.

the field winding, or of the transformer winding, at the corresponding point of the circumference. Only one triangular curve is shown in fig. 56, but by rights there

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ought to be two, displaced to each other—one relating to the field winding and the other to the transformer winding. As long as the iron paths are not saturated, the same triangular curve which represents the distribution of the M.M.F. of the field winding around the armature also represents the distribution of the field *flux* around the armature. The apexes of the two triangles representing the field winding lie on the x-axis while the two triangles representing the transformer winding have their apexes on the y-axis. We will assume that the M.M.F. due to the armature winding (III. in fig. 55) is also distributed around the armature in a triangular manner, the apexes of the respective triangles being on the y-axis. Now, the resultant of the two sets of triangular curves in the direction of the y-axis is again a triangular curve, because the triangles relating to I. and III. have the same base. Of course the currents in I. and III. need not be in phase with one another.

With a triangular distribution, the maximum value of the M.M.F. is twice the average M.M.F. The same may be said of the flux density unless the iron parts are highly saturated.

While for a sinusoidal distribution $M_x = Q\,B_{max.}/1{\cdot}57$, we find for a triangular distribution $M_x = Q\,B_{max.}/2$. The effective 'rotation voltage' induced by the field flux between the brushes at the speed N′ is now

$$E_x = \frac{4}{\sqrt{2}}\, s\,N'M_x\ 10^{-8}, \text{ in which } M_x = 0{\cdot}5\ Q\,B_{max.}$$

The effective voltage induced in the armature winding by the transformer flux, assuming the latter to link with *all* the armature turns, is

$$= 4{\cdot}44\ s\,N\,M_y\ 10^{-8}, \text{ in which } M_y = 0{\cdot}5\ B_{y\,max.}$$

But the armature winding is not concentrated but distributed. This leads—bearing the triangular distribution in mind—to a reduction of the effective E.M.F. in the ratio of three to two.[1]

Hence the effective voltage

$$E_y = \frac{4{\cdot}44}{1{\cdot}5}\, s\, N\, M_y\, 10^{-8} = 2{\cdot}95\, s\, N\, M_y\, 10^{-8}.$$

Thus in the case of a triangular distribution the principal equation of the repulsion motor takes the form

$$2{\cdot}95\ N\, M_y = 2{\cdot}82\ N'M_x \quad \text{or} \quad 1{\cdot}045\ N\, M_y = N'M_x.$$

At synchronous speed, therefore,

$$M_y = 0{\cdot}955\ M_x.$$

The following E.M.F.'s are induced in one armature turn undergoing commutation:—

$$E_w = 4{\cdot}44\ N\, M_x\, 10^{-8}, \text{ and}$$

$$E_a = \frac{4}{\sqrt{2}}\, 2\ N'M_y\, 10^{-8} = 5{\cdot}66\ N'M_y\, 10^{-8}.$$

But $M_y = 0{\cdot}955\ M_x$; hence

$$E_a = 5{\cdot}4 \frac{N'^2}{N}\, M_x\, 10^{-8}.$$

[1] In the case of the sinusoidal distribution the E.M.F. was reduced in the ratio of 1·57 to 1. With a triangular distribution this ratio is 3/2, as may be shown as follows:—Multiply each ordinate of the triangular curve with its corresponding abscissa and plot a new curve with these products as ordinates, such ordinates being evidently proportional to the square of the respective abscissæ. The area enclosed between this new curve and the axis of abscissæ is equal to 1/3 × total length of abscissa × maximum ordinate. If, on the other hand, the stator winding were of such a form that the *total* flux produced by it would link with *all* the armature turns, then the corresponding area would be equal to 1/2 × total length of abscissa × maximum ordinate. Hence the

$$\text{reduction-factor} = \tfrac{1}{2}/\tfrac{1}{3} = 3/2.$$

E_w and E_a are opposed to one another, the resultant being

$$E_w - E_a = 4{\cdot}44\ N\ M_x\ 10^{-8} \times \left[1 - 1{\cdot}22\left(\frac{N'}{N}\right)^2\right].$$

While for a sinusoidal distribution of the flux around the armature and at synchronous speed

$$E_w - E_a = 0,$$

we now find that in the case of the distribution being triangular and at synchronous speed

$$E_w - E_a = -0{\cdot}22 \times 4{\cdot}44\ N'M_x\ 10^{-8} = -0{\cdot}22\ E_w.$$

This means—leaving the reactance voltage out of consideration—that the E.M.F.'s in the commutating coil balance each other not at synchronous speed as before, but at a speed about 10 per cent. lower than synchronous speed. There is now no speed at which the armature iron losses disappear completely.

So far, we have assumed that the M.M.F.'s of the windings had only to force the fluxes across the air-gap. As soon as the teeth become highly saturated, however, the fluxes along the air-gap are no longer distributed in a triangular manner. The curve representing the distribution in such cases is something between a triangle and a sine curve. This is shown in fig. 57 in which I. is the triangle, III. the sine curve, and II. the curve which obtains mostly in practice. When the teeth are very highly saturated curve II. may even lie below curve III.

In the general equation

$$E_w - E_a = 4{\cdot}44\ N'M\ 10^{-8}\left(1 - k\left(\frac{N'}{N}\right)^2\right),$$

k is a coefficient which varies in accordance with the shape of curve II., *i.e.* with the degree of saturation. The value of k must be estimated by the designer in any

particular case. For a sinusoidal distribution $k = 1$, and for a triangular distribution $k = 1{\cdot}22$.

When the teeth are saturated, the flux M is no longer

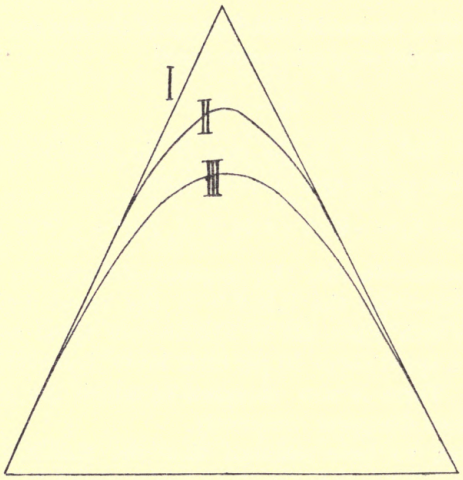

FIG. 57.—Curves indicating the different kinds of flux distribution around the armature.

given by the equation $M = 0{\cdot}5\ Q\,B_{max.}$, but may be determined from the general formula

$$M = \frac{Q \times B_{max.}}{k'},$$

in which $k' = 2$ for a triangular distribution (non-saturated teeth) and $k' = 1{\cdot}57$ for a sinusoidal distribution.

The ratio $\dfrac{\text{A.T. for air-gap}}{\text{A.T. for (air-gap + iron)}}$ is an approximate criterion of the kind of distribution along the air-gap. If this ratio is nearly unity, the distribution is nearly triangular, and if its value is 2/3 then the distribution is approximately sinusoidal.

Of course the field flux and the transformer flux are not always of the same magnitude, and the saturation may differ in consequence in the two cases. It may happen, for instance, that $k' = 1{\cdot}6$ in respect to the field flux, while at the same time $k' = 1{\cdot}9$ may be the proper value with regard to the transformer flux.

These considerations and conclusions are modified to a certain extent by the fact that the M.M.F.'s of the two windings in question are superposed. It would not be quite correct to say that the fluxes, as considered separately, are superposed since the saturation of the teeth makes a simple addition of the two overlapping fluxes impossible. The percentage error is not likely to be great in repulsion motors, however, because in these motors high densities at normal load are avoided as much as possible.

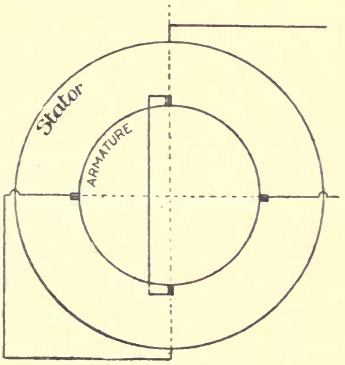

FIG. 58.

Our considerations were based on the kind of winding illustrated diagrammatically in fig. 55, *i.e.* on a winding such as will hardly ever be employed. But all the formulæ relating to a motor provided with the type of winding shown in fig. 55 are at once applicable to motors having the equivalent winding of fig. 58. This figure represents a compensated repulsion motor provided with a d.c. winding on the stator.

The usual kind of repulsion motor is shown diagrammatically in fig. 59. It is an illustration of one of those cases in which the simplicity of the mechanical design

and arrangement renders a theoretical investigation exceedingly difficult. A simple solution of the problem

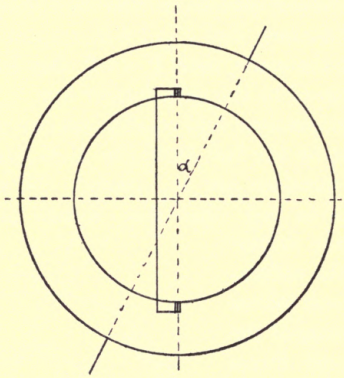

FIG. 59.

of determining the distribution of the flux along the

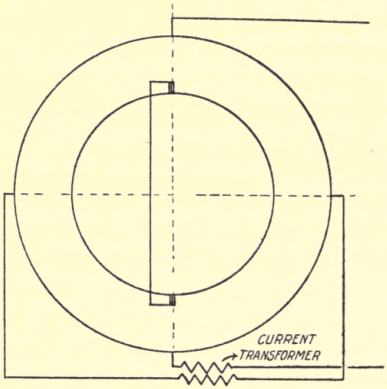

FIG. 60.

gap is not easy to find since this distribution can neither be represented by a triangle nor a sine curve. The

distribution cannot be arrived at by superposing the windings I. and II. of fig. 55.

By adding a current transformer to the arrangement of fig. 55 we get the diagram of fig. 60, only one d.c. stator winding now being required. Such an arrangement would be equivalent to that of fig. 55 but by no means to that of fig. 59. The reason is that a d.c. winding corresponds to a triangular distribution of the M.M.F. and not to a sinusoidal distribution. While the resultant of two sine waves of equal length, even if displaced

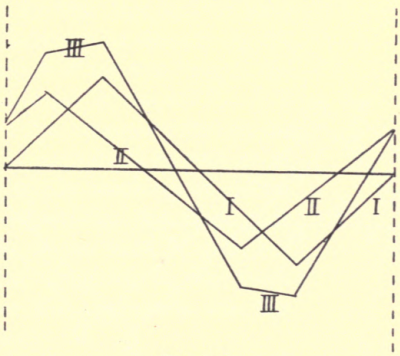

FIG. 61.

in space, is again a sine curve, the resultant curve of two triangles which, although having the same length of base, are displaced in space is no longer a triangle. This is a fundamental difference. In fig. 61, for instance, the resultant of the two triangles I. and II. is the irregular broken line III.

Thus, if we may not resolve the M.M.F. into components, we will have to consider the action of the armature upon the stator as a whole.

In fig. 62, A D B D′C represents the M.M.F. of the stator along the air-gap while E F G H J K indicates the

distribution of the armature–M.M.F. Both graphs are displaced in space by a distance a corresponding to an angular displacement α of the brushes (fig. 59). The ordinates of curve II. are in phase with the armature current J_2, those of curve I. being in phase with the primary current J_1.

We know from previous explanations that the primary current is displaced in phase by nearly 180 degrees from the armature current. In fig. 62, however, one of the graphs has been plotted with the negative values as

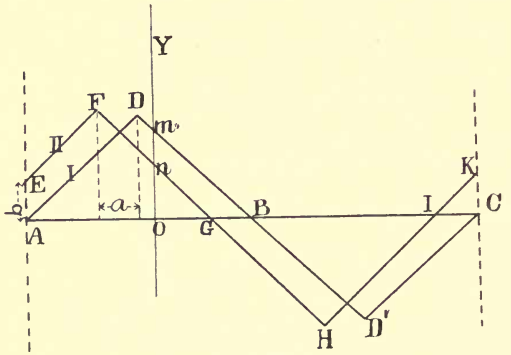

FIG. 62.

ordinates so that the resultant M.M.F. at any point of the circumference is simply equal to the difference of the ordinates of curves I. and II. at this point. Assuming the currents to vary sinusoidally with the time, the phase of any current or M.M.F. may be represented by a vector of a clock-face diagram. All ordinates of curve I. are in phase with one another; likewise all the ordinates of curve II. are in phase with each other. But the phase of curve I. differs by a certain angle ϕ_{12} from the phase of curve II. This can be represented graphically by assuming that the plane through curve I. forms with the

plane through curve II. this angle ϕ_{12}. These two planes rotate at a uniform speed around the principal axis A C, and they intersect a third plane at right angles to the principal axis in two lines which are the time-vectors (I. and II. in fig. 63) of the two graphs. The determination of the resultant graph is now comparatively simple. For a point O (fig. 62) of the circumference the resultant is found as follows:—

Draw through O the perpendicular O Y, and mark the point *m* on vector I. and *n* on vector II. so that O*m* and O*n* have the same values in fig. 63 as in fig. 62. The difference of the two vectors O*m* and O*n* is equal to $nm = O P$. O P represents the maximum flux at the point O in magnitude as well as in phase. Strictly speaking, of course, *mn* only represents the magnitude and phase of the M.M.F. at O, but at low flux densities in the teeth this vector also represents the flux density at O.

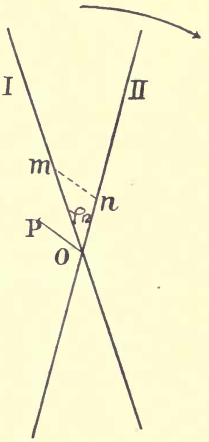

FIG. 63.

In like manner the magnitude and phase of the M.M.F. is found for any other point of the circumference, providing that the currents J_1 and J_2 and the phase-difference, ϕ_{12}, between them are known.

In fig. 64, O Q and O Q′ represent the maximum positive and the maximum negative value of the M.M.F. of the stator winding; similarly O R and O R′ represent the maximum values of the M.M.F. of the armature winding at either side of O. A point rotating at uniform speed around the air-gap corresponds to a point travelling at constant speed along the line A C in fig. 62. And this movement of one point along the air-gap corresponds, in

fig. 64, to the movements of two points—one of which oscillates between Q and Q′, the other oscillating between R and R′. O on vector I. (fig. 64) corresponds to A in fig. 62, and the point travels from Q to Q′ in the same time that it takes the point on vector II. to move from R to R′. The two points do not pass through O at the same moment; the point on vector I. is always in advance by the distance $b = EA$ (fig. 62). The movements of these two points suffice to represent the distribution of

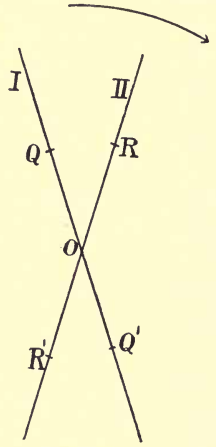

FIG. 64.

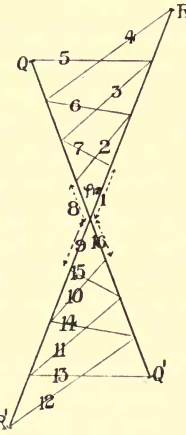

FIG. 65.

the M.M.F. with regard to time, magnitude, and place. The point along the air-gap is fixed by the instantaneous positions of the two points; the maximum M.M.F. which occurs at this place is represented by the distance between these two points, while the phase, finally, of the M.M.F. is given by the direction of the connecting line, the line being drawn from the point on II. to the point on I. In order to make this graphical representation quite clear, fig. 65 has been drawn representing the case

in which the point on II. leads the point on I. by a distance equal to one-quarter of O R. The connecting lines are drawn for every one-sixteenth of a revolution, the numbers indicating the order of succession. It is easy to see how the flux varies in magnitude and phase from point to point around the circumference for given values of J_1, J_2, and $\cos \phi_{12}$.

This diagrammatic representation may be used in connection with the design of a repulsion motor as follows:—

The repulsion motor is first calculated—say, for synchronous speed—by assuming approximate values of the coefficients k and k'. One obtains then the armature current in magnitude and phase in the manner indicated in our previous example. Now it is possible to check k and k' backwards by determining the kind of flux distribution around the armature.

We will try, however, to determine the E.M.F.'s in the armature and stator windings directly from figs. 64 and 65.

Referring to fig. 66, in which *mn* represents the flux at one particular point along the air-gap, it will be understood that this flux induces in the stator winding an E.M.F. lagging 90 degrees behind *mn*. In magnitude this E.M.F. is proportional to the distance *mn* and to the distance O*m*, the latter distance representing the number of stator turns which are linked with the flux *mn*. Thus the E.M.F. induced in the stator winding is proportional to $(Om \times mn)$ and its vector lags 90 degrees behind *mn*. The total E.M.F. induced in the stator winding is obviously equal to the geometrical sum of all these voltage vectors around the armature. The summation is made easier by resolving the E.M.F. induced at every point around the armature into a watt-component and a wattless component. In fig. 66, for

instance, the component of the E.M.F. (at the point corresponding to mn) in phase with the stator current is proportional to the product

$$Om \times mn \times \sin(nmO),$$

hence also proportional to the area of the triangle mOn. By adding all the areas mOn corresponding to all the positions of mn, we obtain the watt-component of the total E.M.F. induced in the stator winding.

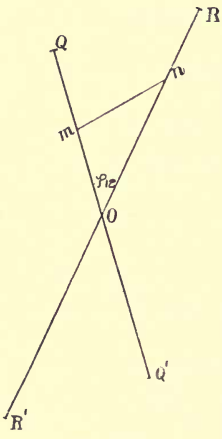

FIG. 66.

The area of triangle

$$mOn = \tfrac{1}{2} \times Om \times On \times \sin \phi_{12}.$$

Thus we arrive at the conclusion: —The watt-component of the E.M.F. corresponding to the position mn is proportional to

$$Om \times On \times \sin \phi_{12}.$$

Similarly, the wattless component corresponding to the position mn is proportional to

$$Om^2 - Om \times On \times \cos \phi_{12},$$

and the watt-component of the total E.M.F. induced in the stator is proportional to

$$\Sigma\,(Om \times On \times \sin \phi_{12}),$$

and the wattless component of this total E.M.F. is proportional to

$$\Sigma\, Om^2 - \Sigma\,(Om \times On \cos \phi_{12}).$$

The E.M.F. in the armature may also be resolved into two components, one being the watt-component in phase with the armature current and equal to

$$\Sigma\, h \times (Om \times On \times \sin \phi_{12}),$$

and the other being the wattless component displaced in phase by 90 degrees from the armature current and equal to

$$\Sigma\, On^2 - h\,\Sigma\,(Om \times On \times \sin \phi_{12}).$$

All these expressions furnish relative values, h being a coefficient introduced to put the relative values on the same basis. This coefficient h is easily calculated, for, according to the law of the conservation of energy,

Stator current × watt-component of the E.M.F. induced in the stator = Armature current × watt-component of the E.M.F. induced in the armature.

Hence $$h = \frac{\text{stator current}}{\text{armature current}} = \frac{J_1}{J_2}.$$

The summation of the E.M.F.'s induced in the stator and armature windings may be carried out by reference to fig. 66.

Let λ denote the ratio of the lead of one of the oscillating points to its maximum amplitude. It is then found that the watt-component of the E.M.F. induced in the stator winding is proportional to

$$(\tfrac{4}{3} - 2\lambda^2 + \tfrac{2}{3}\lambda^3) \sin \phi_{12} \quad . \quad . \quad . \quad . \quad . \quad (1);$$

similarly, the wattless component is proportional to

$$\tfrac{4}{3} - (\tfrac{4}{3} - 2\lambda^2 + \tfrac{2}{3}\lambda^3)\ \frac{\cos \phi_{12}}{h} \quad . \quad . \quad . \quad (2).$$

Both values are also proportional to the stator current. Further, the watt-component of the E.M.F. induced in the armature winding is proportional to

$$(\tfrac{4}{3} - 2\lambda^2 + \tfrac{2}{3}\lambda^3) \sin \phi_{12} \quad . \quad . \quad . \quad . \quad . \quad (3),$$

and the wattless component is proportional to

$$\tfrac{4}{3}h - (\tfrac{4}{3} - 2\lambda^2 + \tfrac{2}{3}\lambda^3) \cos \phi_{12} \quad . \quad . \quad . \quad (4).$$

These last two components are also proportional to the armature current.

The absolute values are easily obtained by introducing the magnetising current J_0, defined as that current which induces in the stator winding (the armature being open) an E.M.F. equal to that obtaining at full-load, *i.e.* = terminal voltage minus the ohmic drop in voltage.

In this case, as we already know,

$E = \frac{2}{3} \times 4{\cdot}44 \times N \times s \times M \times 10^{-8} = 2{\cdot}95\ NsM\ 10^{-8}$, and
$M = \frac{1}{2} \times Q\, B_{max.}$

The requisite value of $B_{max.}$ is obtained from these equations, and it is now an easy matter to calculate the current J_0 necessary to produce this flux.

If $\lambda = 1$, *i.e.* if the direction of the brushes is perpendicular to the axis of the stator winding, it is gathered from equations (1) and (2) that there is no watt-component while the wattless component assumes a value proportional to

$$\tfrac{4}{3} \times \text{stator current.}$$

This enables us to determine the constant of equations (1) to (4).

Since the magnetising current J_0 corresponds to the terminal voltage E, we may write equation (2) for $\lambda = 1$ in the following form:—

$$E = \text{constant} \times \tfrac{4}{3} \times J_0.$$

Hence the 'constant' is $= \dfrac{E}{\frac{4}{3} \times J_0}$.

Introducing this constant into equations (1) to (4) we get:

Wattless component of the stator voltage

$$= E\left[\frac{J_1}{J_0} - \frac{J_2}{J_0}(1 - 1{\cdot}5\lambda^2 + 0{\cdot}5\lambda^3) \cos \phi_{12}\right] \quad . \quad . \quad (5).$$

Watt-component of the stator voltage

$$= E\left(\frac{J_2}{J_0}\right)(1 - 1{\cdot}5\lambda^2 + 0{\cdot}5\lambda^3) \sin \phi_{12} \quad . \quad . \quad . \quad . \quad (6).$$

Wattless component of the armature voltage

$$= E\left[\frac{J_2}{J_0} - \frac{J_1}{J_0}(1 - 1{\cdot}5\lambda^2 + 0{\cdot}5\lambda^3) \cos \phi_{12}\right] \quad . \quad . \quad (7).$$

Watt-component of the armature voltage

$$= E\left(\frac{J_1}{J_0}\right)(1 - 1{\cdot}5\lambda^2 + 0{\cdot}5\lambda^3) \sin \phi_{12} \quad . \quad . \quad . \quad . \quad (8).$$

In these equations the armature current J_2 refers to the case in which the stator and the armature have the same number of turns.

Let, for brevity,

$$1 - 1{\cdot}5\lambda^2 + 0{\cdot}5\lambda^3 = x.$$

By squaring (5) and (6) and adding, we get

$$E^2 = E^2\left[\left(\frac{J_1}{J_0}\right)^2 + \left(\frac{J_2 x}{J_0}\right)^2 - 2\frac{J_1}{J_0}\frac{J_2 x}{J_0} \cos \phi_{12}\right]; \text{ hence}$$

$$J_0^2 = J_1^2 + (xJ_2)^2 - 2xJ_1J_2 \cos \phi_{12} \quad . \quad . \quad . \quad (9).$$

This means that the current J_0 is equal, in magnitude, to the geometrical sum of J_1 and $x J_2$

From equations (7) and (8) we get

$$E_2^2 = E^2\left[\left(\frac{x J_1}{J_0}\right)^2 + \left(\frac{J_2}{J_0}\right)^2 - \frac{2xJ_1}{J_0}\frac{J_2}{J_0} \cos \phi_{12}\right]; \text{ hence}$$

$$\left(\frac{J_0 E_2}{E}\right)^2 = (x J_1)^2 + J_2^2 - 2(xJ_1)J_2 \cos \phi_{12} \quad . \quad . \quad (10),$$

in which E_2 is the E.M.F. induced in the armature winding. It is thus seen that the geometrical addition of xJ_1 and J_2 is equal, in magnitude, to $\frac{E_2 J_0}{E}$.

In addition to the E.M.F.'s induced by the pulsations of the flux, there appears in the armature also a voltage due to rotation. This voltage can be calculated by reference to fig. 66. It varies from point to point along the air-gap; at a given point it is proportional to and in phase with the vector $m\,n$. We resolve the total E.M.F. of rotation into a component in phase with the armature current and into a component at right angles to it. It will be found that the former component is proportional to

$$\Sigma\, Om \cos \phi_{12} - \Sigma\, On,$$

while the wattless component is proportional to

$$\Sigma\, Om \sin \phi_{12}.$$

Carrying out the summation, it is found that the watt-component is proportional to $(3\lambda^2 - 6\lambda) \sin \phi_{12}$, while the wattless component is proportional to

$$(3\lambda^2 - 6\lambda) \cos \phi_{12}.$$

These relative values already indicate that the E.M.F. induced in the armature by rotation differs in phase by the angle ϕ_{12} from the armature current; in other words, the E.M.F. referred to is exactly in phase with the stator current.

In order to obtain the absolute value of this E.M.F., we calculate it for $\lambda = 1$, and get

$$\frac{4}{\sqrt{2}}\, s\, N' M_x\, 10^{-8} = 2{\cdot}82\, s\, N' M_x\, 10^{-8}.$$

We know that the no-load current J_0 induces in the stator winding the voltage

$$E = 2{\cdot}95\, s\, N\, M_x\, 10^{-8};$$

hence, assuming the number of turns of the stator and

armature windings to be the same, we get for the 'rotation voltage' in the armature

$$0{\cdot}955\ \frac{EN'}{N}.$$

For any other current J_1, the voltage becomes

$$0{\cdot}955\ \frac{N'}{N}\ \frac{J_1}{J_0}\ E;$$

and, finally, for any other value of λ, the voltage in question is

$$(\lambda^2 - 2\lambda)\ 0{\cdot}955\ \frac{N'}{N}\ \frac{J_1}{J_0}\ E.$$

When the brushes are short-circuited, the sum-total of all the E.M.F.'s in the armature winding is zero. Therefore, neglecting the drop of voltage in the short-circuit,

$$\lambda^2 - 2\lambda)\ 0{\cdot}955\ \frac{N'}{N}\ \frac{J_1}{J_0}\ E\cos\phi_{12} + E\frac{J_1}{J_0}\ (1 - 1{\cdot}5\lambda^2 + 0{\cdot}5\lambda^3)\sin\phi_{12} = 0.$$

Hence
$$\tan\phi_{12} = 0{\cdot}955\ \frac{\lambda^2 - 2\lambda}{1 - 1{\cdot}5\lambda^2 + 0{\cdot}5\lambda^3}\ \frac{N'}{N} \quad . \quad . \quad . \quad . \quad (11).$$

Also, in absolute values,

$$(\lambda^2 - 2\lambda)\ 0{\cdot}955\ \frac{N'}{N}\ \frac{J_1}{J_0}\ E = E\sqrt{\left(\frac{J_2}{J_0}\right) + \left(\frac{J_1\, x}{J_0}\right)^2 - 2x\ \frac{J_1 J_2}{J_0^2}\cos\phi_{12}}.$$

Hence

$$J_2^2 + x\, J_1^2 - 2\, x\, J_1\, J_2\cos\phi_{12} = \left[(2\lambda - \lambda^2)0{\cdot}955\ \frac{N'}{N}\ J_1\right]^2 \quad . \quad . \quad (12).$$

Equations (9) to (11) are the principal equations of the repulsion motor.

Let
$$\beta = (2\lambda - \lambda^2)\ 0{\cdot}955\ \frac{N'}{N}.$$

It is gathered from equation (12) that J_2, xJ_1, and βJ_1 form a triangle in which the angle between J_2 and xJ_1 is equal to the phase difference ϕ_{12} between the stator current and the armature current.

If, therefore, O′A in fig. 67 is drawn to represent J_1, and $OA = xJ_1$ and $AB = J_2$, then OB represents βJ_1 in magnitude.

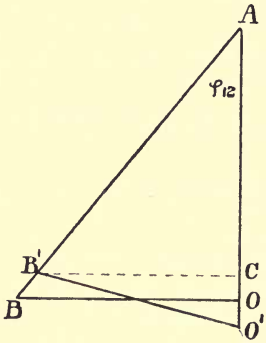

FIG. 67.

From equation (11)

$$\tan\phi_{12} = \frac{\beta}{x} = \frac{\beta J_1}{xJ_1} = \frac{OB}{OA}.$$

This, however, is only possible if OB is perpendicular to OA.

Let $AB' = xAB = xJ_2$, then, according to equation (9), the three vectors $O'A = J_1$, $AB' = xJ_2$, and $O'B' = J_0$ form a triangle. O′B′ in fig. 67 thus represents the magnetising current in the stator. The terminal voltage E leads O′B′ by 90 degrees, and is proportional to O′B′.

According to equation (10)

$$OB = \frac{J_0 E_2}{E} = \frac{O'B' \times E_2}{E}, \quad \text{hence} \quad \frac{OB}{E_2} = \frac{O'B'}{E}.$$

It is thus evident that the relation between OB and the armature voltage is the same as that between O′B′ and the terminal voltage. This means that OB and O′B′, measured with a common scale, give directly the armature voltage and the terminal voltage respectively, while the angle enclosed by these two vectors is equal to the phase difference between the terminal voltage and the current.

Draw B′C perpendicular to O A, then

$$\tan\phi = \tan\ (\mathrm{O'B'C}) = \frac{\mathrm{O'C}}{\mathrm{B'C}} = \frac{\mathrm{A\,O' - C\,O'}}{\mathrm{B'C}} = \frac{\mathrm{J_1 - x^2 J_1}}{x\beta\,\mathrm{J_1}} = \frac{1-x^2}{x\beta}.$$

From the table on p. 175 it is gathered that at synchronous speed

$$\beta^2 + x^2 = 1 \text{ (very nearly).}$$

Hence, in this case, $\tan\phi = \frac{\beta^2}{x\beta} = \frac{\beta}{x}$. But it was shown above that $\tan\phi_{12} = \frac{\beta}{x}$. Therefore, at synchronous speed, $\phi = \phi_{12}$.

If α denote the angle between the primary axis and the secondary axis, then, approximately, $\frac{\beta}{x} = \tan\alpha$—*i.e.* at synchronous speed,

$$\phi = \phi_{12} = \alpha.$$

At other speeds we have, approximately,

$$\tan\phi = \tan\alpha\,\frac{\mathrm{N}}{\mathrm{N'}} \quad \text{and} \quad \tan\phi_{12} = \tan\alpha\,\frac{\mathrm{N'}}{\mathrm{N}}\text{; hence}$$

$$\tan\phi = \tan\phi_{12}\left(\frac{\mathrm{N}}{\mathrm{N'}}\right)^2.$$

In the diagram (fig. 67) account has been taken neither of the primary nor secondary leakage. In order to do this we assume that only v' × stator-A.T. have an effect upon the armature and that only v'' × armature-A.T. influence the stator. On this assumption the vectors affecting the armature are $\mathrm{J_2}$ and $v'x\mathrm{J_1}$, the resultant of which is $\beta\,\mathrm{J_1}$; similarly, the stator if affected by the quantities $\mathrm{J_1}$ and $v''x\mathrm{J_2}$, the resultant of which is $\mathrm{J_0}$.

In fig. 68, O′A represents the stator current $\mathrm{J_1}$ and A B the armature current $\mathrm{J_2}$ ($\mathrm{J_2}$ is reduced to the same

number of turns as pertains to J_1), and we find that

$$OA = x v' J_1, \qquad AC = x^2 v' v'' J_1, \qquad AB' = x v'' J_2,$$
$$OB = \beta J_1, \qquad O'B' = J_0, \text{ and}$$
$$B'C \text{ (which is at right angles to } O'A) = \beta x v'' J_1.$$

Obviously $\dfrac{B'C}{B'O'} = \dfrac{\text{watt-component of term. volt.}}{\text{terminal voltage}} = \cos\phi.$

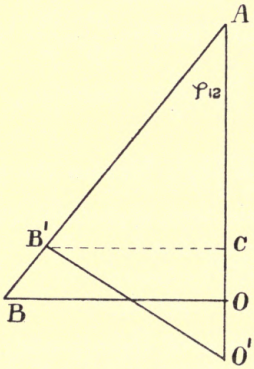

FIG. 68.

Hence $$\cos\phi = \frac{\beta x v'' J_1}{J_0}, \quad \text{or}$$

$$\tan\phi = \frac{O'C}{B'C} = \frac{(1 - v'v''x^2)}{\beta x v'' J_1} J_1 = \frac{1 - v'v''x^2}{\beta x v''}.$$

For normal motors $v' = v'' = v$, and if vx be denoted by q, we can write

$$\tan\phi = \frac{1 - q^2}{\beta q}.$$

The ohmic drop of voltage was neglected in the dia gram fig. 68. It can be taken into account by proceeding along the lines indicated in previous chapters; it is unnecessary to repeat this simple operation here again.

v is given by the equation $v = 1 - \frac{\sigma_k}{2}$.

$$x = 1 - 1{\cdot}5\,\lambda^2 + 0{\cdot}5\,\lambda^3.$$

$$\beta = (2\lambda - \lambda^2)\,0{\cdot}955\,\frac{N'}{N}.$$

The diagrams which have been thus developed are only strictly correct for non-saturated motors, but they give fair results—accurate enough for most practical purposes—for densities in the teeth up to 18,000 or 20,000 lines per sq. cm.

The simple diagram, fig. 66, enables us to determine at which point along the air-gap the maximum flux density occurs.

Obviously, the absolute maximum value of $m\,n$ occurs in one of the following four positions:—

(1) m in O (position D in fig. 69).
(2) n ,, O (,, C ,,).
(3) m ,, Q (,, B ,,).
(4) n ,, R (,, A ,,).

Whether (1) is larger than (2) or *vice versâ* depends on the relative values of J_1 and J_2. If J_2 is larger than J_1, then mn is larger in position (1) than in position (2) and *vice versâ*. Similarly, mn is larger in position (4) than in position (3) when J_2 is larger than J_1.

Thus the two corresponding maxima of flux-density in the air-gap of a respulsion motor are not displaced by an angle of 90 degrees, as might be thought, but by an angle $90 \times (1 \pm \lambda)$ degrees.

From fig. 68 $J_2 = J_1 \sqrt{\beta^2 + q^2}$;

i.e. for speeds lower than synchronous speed $J_2 < J_1$,
and ,, higher ,, ,, $J_1 < J_2$.[1]

[1] Strictly speaking, J_1 is larger than J_2 over just a small range of speed beyond synchronous speed.

It may also be learned from fig. 66 that the vector *mn*, for O R > O Q, is larger in position (4) than in position (1).

Thus it is seen that the maximum flux density occurs at the points B or C (fig. 69) for speeds below synchronism, and at the point A at speeds higher than synchronous speed.

A close investigation will show that the maximum

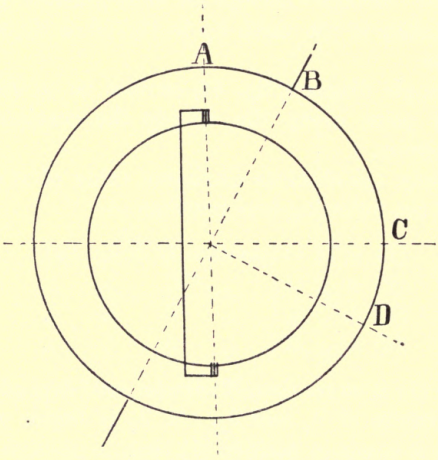

FIG. 69.

flux density occurs at C at all speeds up to a certain speed, which depends on the angle α; between this speed and synchronous speed the maximum flux density is at B, while beyond the last-named speed A is the point at which the maximum density occurs.

The following table gives a number of quantities calculated for various values of α and λ:—

$\alpha = \lambda \times 90$ Degrees =	5 Degrees.	10 Degrees.	15 Degrees.	20 Degrees.	25 Degrees.	30 Degrees.	35 Degrees.	40 Degrees.	45 Degrees.
$2\lambda - \lambda^2$, ...	0·107	0·208	0·306	0·394	0·477	0·555	0·63	0·69	0·75
$x = 1 - 1{\cdot}5\lambda^2 + 0{\cdot}5\lambda^3$, . .	0·995	0·982	0·960	0·930	0·896	0·852	0·80	0·745	0·69
$\tan \phi_{12}$ for $N = N'$, . .	0·102	0·202	0·308	0·405	0·508	0·62	0·75	0·885	1·02
ϕ_{12} for $N = N'$ in degrees,	5·8	11·4	17·1	22·0	26·9	31·8	36·8	41·5	45·7
$\sin \phi_{12}$ for $N = N'$, . .	0·102	0·194	0·295	0·375	0·455	0·53	0·60	0·66	0·72
$\sqrt{\beta^2 + x^2}$ for $N = N'$, . .	1	1	1	1·01	1·01	1·01	1·01	1·01	1·01

It follows from this table that we may write with sufficient accuracy $x = \cos \alpha$ and $\beta = \sin \alpha \frac{N'}{N}$.

APPENDIX II.

SOME OSCILLOGRAPHIC TESTS ON THE COMMUTATION OF SINGLE-PHASE MOTORS.

It was explained in Chapter VI. that the coils undergoing commutation react on the field, the value of this reaction being

$$0{\cdot}083\, n^2\, \frac{E_f\, z}{W} \text{ ampere-turns.}$$

The existence of this reaction may be verified experimentally by measuring the short-circuit current of d.c. dynamos for different brush-widths. Such tests were carried out by the author, who found—contrary to what might be expected on first thoughts—that the short-circuit current of the dynamo gradually diminished as the brush-width was increased. In the light of the theory expounded in Chapter VI., however, this behaviour is readily understood; the fact is that the reaction on the field, being proportional to n^2 (n = number of segments covered by the brush), greatly increases with the width of the brush. By doubling the width, for instance, the reaction is increased fourfold. In consequence, the M.M.F. of the field is greatly reduced, so that the short-circuit current grows smaller in spite of the reduced ohmic resistance of the circuit.

Another experimental proof of the theory in Chapter VI. may be deduced from the oscillograms published by Mr F. Creedy in his Paper entitled "The Alternating-current Series Motor," and read before the Institution of Electrical Engineers on 13th April 1905. Mr Creedy has kindly placed at our disposal a number of unpublished oscillograms taken at the same time as those

illustrating his Paper. In order to understand and appreciate these oscillograms fully, we reproduce in fig. 70

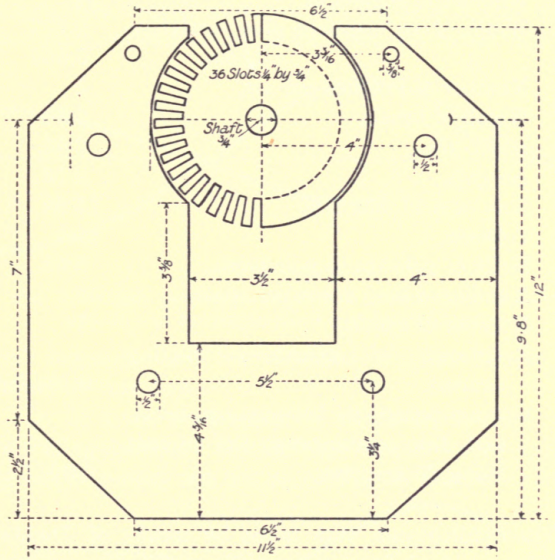

FIG. 70.—Sketch of experimental motor.

a sketch of the experimental motor used. The main dimensions of this motor, in centimetres, are :—

Diameter of armature, . . .	13·3
Number of slots, ...	36
Dimensions of slot, ...	1·9 × 0·63
Internal diameter of armature stampings, ...	1·9
Total length of armature, . . .	9·7
Effective „ „ . . .	8·7
Number of conductors per slot, .	14
Number of turns in series between the brushes, ...	126

Diameter of commutator, . . .	7·3
Length „ „ . . .	4·9
Number of segments, . . .	36
Width of one segment plus insulation,	0·637
„ „ without „	0·560
Number of brushes per set, . .	1
Width of brush, ...	2·54
Brush-arc, ...	2·63
Air-gap, ...	0·16
Bore of field magnets, . . .	13·62
Pole-arc, ...	11·5
Ratio of pole-arc to pole-pitch, . .	0·55
Number of field turns per spool, .	42
Number of spools, ...	1
Resistance of armature winding (warm), ...	0·294 ohms.
Contact-resistance between one brush and the commutator, the current being 25 amperes, . . .	0·05 „

The oscillograms were taken by means of Duddell's well-known high-frequency oscillograph; one armature coil was cut open, the ends being connected to two slip-rings on the motor shaft. The leads from these slip-rings were conducted to a suitable resistance, the voltage across which was applied at the terminals of the oscillograph.

Curve I. in fig. 71 represents the main current, while curve II. shows the variation of current in one conductor. It is clearly observed from the latter curve in what manner the current changes during the period of commutation. The four steps in the commutation curve result from the successive entrances of the segments under the brush during the time the coil under study is commutating. Each step comprises a secondary ripple, this being due to the fact that the brush covers a little more

than four segments. By plotting out to scale the successive relative positions of brush and segments, it is seen that, shortly after the first segment enters under the brush, contact is broken between the brush and the

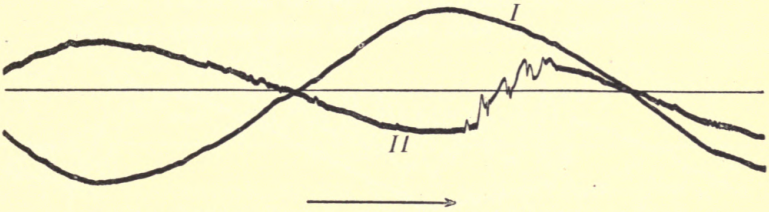

FIG. 71.—Main current curve (I), and current in an armature coil (II), of the single-phase experimental motor.

sixth segment. All the secondary ripples can be explained in this way.

Fig. 72 is an oscillogram in which three steps only are distinguishable. During this test, it would appear, the brush-contact was not as good as in the other tests.

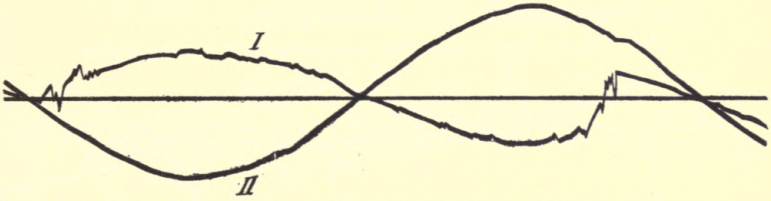

FIG. 72.—Curve I represents the current in the coil under study, while curve II shows the variation of the alternating terminal voltage in the experimental single-phase motor. Main current = 20 amperes; speed = 1200 revs. per minute; frequency of speed = 20 and frequency of line current = $26\frac{2}{3}$ cycles per second.

Curve II. represents the terminal voltage traced simultaneously with the current curve.

In curve II., fig. 71, and especially in fig. 72, there are a number of ripples other than those just dealt with. They must be mainly attributed to the reaction of the

commutating coils on the field. In order to study this reaction, a search coil was wound on the pole shoe, a record of the E.M.F. induced in it being taken by the oscillograph. Figs. 73 to 76 are the oscillograms of the current in the armature coil and of the E.M.F. in the search coil. In each case the main current was 20

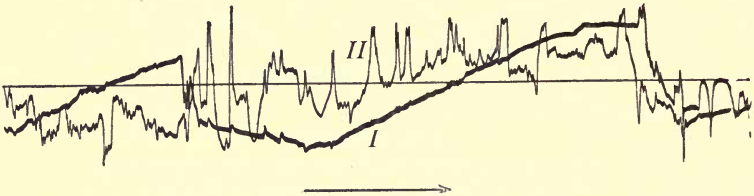

FIG. 73.—Current in coil under study (I), and E.M.F. in search coil (II). Main current = 20 amperes ; speed = 1000 revs. per minute ; frequency of speed = 16·7 and frequency of line current = $26\frac{2}{3}$ cycles per second.

amperes, the frequency of supply being $26\frac{2}{3}$ cycles per second and the speeds being 1000 revolutions per minute (figs. 73 and 74), 1200 revolutions per minute (fig. 75), and 2000 revolutions per minute (fig. 76).

There is no doubt that the comparatively large ripples

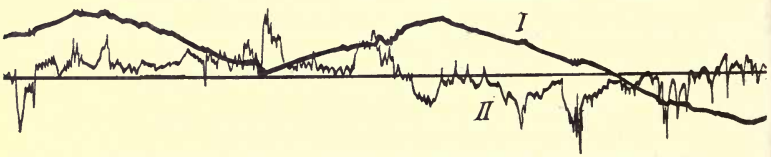

FIG. 74.—Current in coil under study (I), and E.M.F. in search coil (II). Main current = 20 amperes ; speed = 1000 revs. per minute ; frequency of speed = 16·7 and frequency of line current = $26\frac{2}{3}$ cycles per second.

in the E.M.F. curves have their origin in the fluctuating reaction of the coils undergoing commutation. Even if the commutation followed the straight-line law, the reaction would not be constant; it would, as has been mentioned in Chapter VI., p. 46, fluctuate by some 8 per cent. above and below the average for a brush covering

four segments. The total variation, therefore, would be some 16 per cent. of the average value. Since, however, the commutation curve is zigzag to start with, the actual variation will be larger than the figure mentioned.

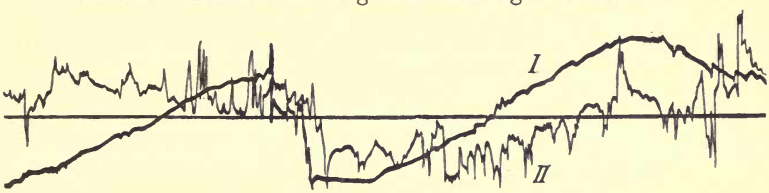

FIG. 75.—Current in coil under study (I), and E.M.F. in search coil (II). Speed = 1200 revs. per minute; frequency of speed = 20 and frequency of supply = $26\frac{2}{3}$ cycles per second.

Applying the formula for the reaction of the commutating coils to this particular case, we get

$$\text{Reaction} = \frac{0{\cdot}083\, n^2 \times z \times E_f}{W} = \frac{0{\cdot}083 \times 4^2 \times 1 \times E_f}{{\cdot}05} = 18{\cdot}5\, E_f.$$

E could be calculated for each of the special cases

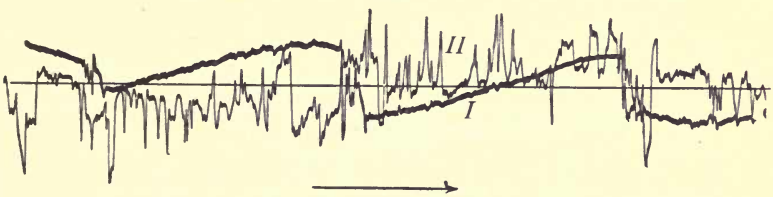

FIG. 76.—Current in coil under study (I), and E.M.F. in search coil (II). Main current = 20 amperes; speed = 2000 revs. per minute; frequency of speed = 33·4 and frequency of line current = $26\frac{2}{3}$ cycles per second.

alluded to, but, as the brush was only partially bearing on the commutator during a part of the tests, such a calculation would not yield accurate results. It is clear, however, that if the machine had a normal sparking-voltage E_f—say, from 3 to 5 volts—the reaction would

amount to from 60 to 90 ampere-turns. When it is considered that the field A.T. during these tests amounted to 840 A.T. only, it is at once seen that we have a valid explanation of the character of the curves II. in figs. 73 to 76.

It will be gathered from fig. 77 that the current in

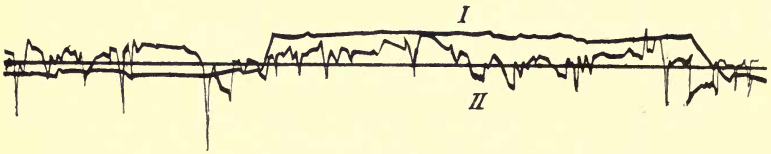

FIG. 77.—Current in coil under study (I), and E.M.F. in search coil (II), the experimental motor being fed with continuous current.

the armature coil under study was larger during one half of a revolution than during the other half. The explanation is furnished by fig. 78, which represents an oscillogram with the brushes lifted off the commutator. It is seen that the armature current has by no means disappeared; this may be due to an out-of-balance of the armature.

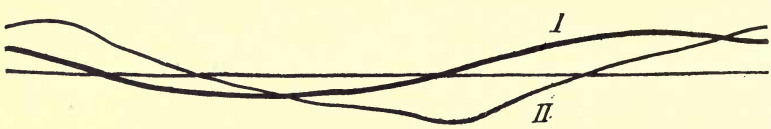

FIG. 78.—Current in coil under study (I), and E.M.F. in search coil (II). The brushes are lifted off the commutator, and the field is excited with 20 amperes.

In one of the tests a search coil of one turn was placed parallel to the pole axis, another coil of one turn being fixed perpendicular to the axis. The E.M.F.'s induced in these coils are shown in figs. 79 and 80. Fig. 79 pertains to alternating-current operation and fig. 80 to continuous-current working.

Curves I. and II. of fig. 81 represent the main current and the E.M.F. of the armature respectively. The motor

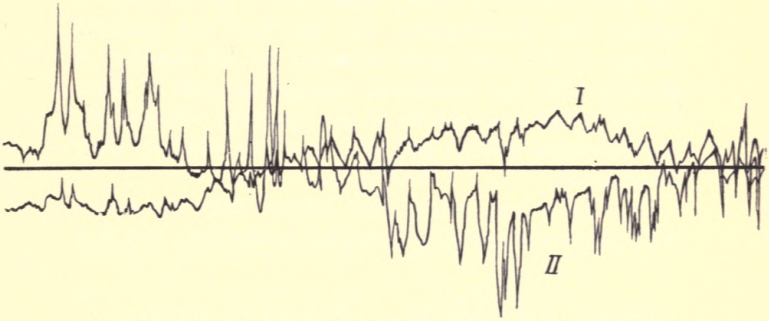

FIG. 79.—E.M.F. in search coil parallel to pole-axis (I), and in search coil at right angle to it (II). Each search coil consisted of one turn. Speed of motor = 1300 revs. per minute.

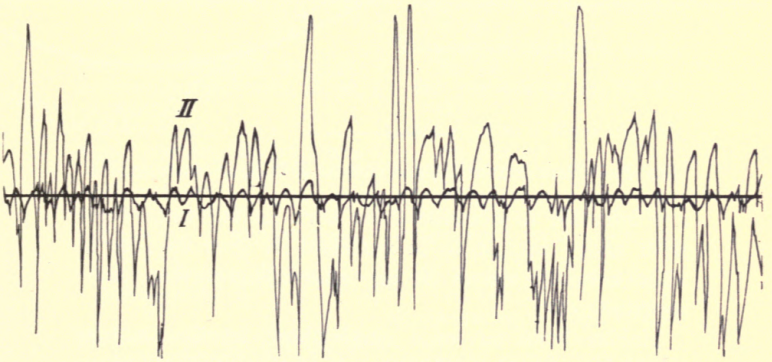

FIG. 80.—E.M.F. in search coil parallel to the pole-axis (I), and in the search coil at right angles to it (II). In this test the motor was running as a continuous current motor, the speed being 1300 revs. per minute, and the current 15 amperes. The coil parallel to the axis consisted of 104 turns.

was driven externally, the main current flowing through the field winding only. The armature was running on

open-circuit excepting for the oscillograph connections. The lag between the E.M.F. across the armature and the field current is clearly observed. Figs. 24 and 25 explain this phenomenon. In this test the sparking voltage in the coils short-circuited by the brushes is identical with the transformer voltage induced in them, for the other components of the sparking voltage, viz. the reactance voltage and the 'rotation voltage,' have disappeared.

Mr Creedy has made some further tests on a d.c. Westinghouse motor of a somewhat larger size; but it is

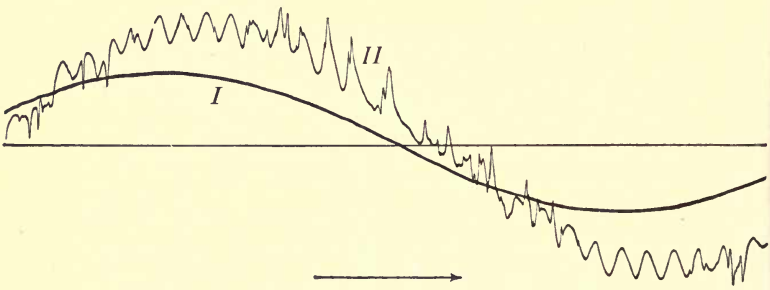

FIG. 81.—Main current (I), and counter-E.M.F. of armature (II).

not intended to describe them here, as the tests dealt with above appear to be sufficient to give an idea of what actually occurs in an alternating-current commutator motor.

Similar tests on large single-phase traction motors would be welcome at the present time. If fluctuations should be observed in them of anything like the same amount as found in Mr Creedy's small experimental motor, means for reducing them would have to be devised.

That this reaction exists in continuous-current motors as well as in alternating-current motors is proved by fig. 77.

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